International macroeconomics

Problems and exercises

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1. Create a table with the following columns (all variables refer to the Spanish economy):

   - Transaction

   - CA
     1. Goods (TB)
     2. Services (TB)
     3. Primary income (Y_1)
        (a) From work (Y_1^w)
        (b) From investment (Y_1^z)
     4. Secondary income = net unilateral transfers (Y_2)
   - KA
     1. Capital transfers
     2. Non-produced, non-financial assets
   - FA
     1. Direct investment (DIB)
     2. Portfolio investment (PIB)
     3. Other investment (OIB)
        (a) Trade credits (LIB)
        (b) Loans (LIB)
        (c) Money (MB)
     4. Official reserves (RES)

   - $CA + KA$
   - Δe_t^{HF}
   - Δb_t^{HF}
   - Δm_t^{HF}
   - $\Delta b_t^{\bar{\text{H}}\bar{\text{F}}}$

   - Y
   - Y^{E}
   - CA

   - Y^{P}
   - Y^{E}
   - TB

Now use the table to indicate how the following transactions affect the variables in each column. Use $\oplus$ if a variables is positively affected and $\ominus$ if it is negatively affected.

1. A Spanish firm in Madrid sells a new car to a Spanish resident in Rome.
2. A Spanish resident in Madrid sells a car to a French resident in Madrid. Distinguish the following cases:
   (a) The Spanish resident is a firm and the car is new.
   (b) The Spanish resident is a firm and it's a second-hand car.
   (c) The Spanish resident is a household and it's a second-hand car.
3. A Spanish resident buys shares of a Spanish firm from a German resident.
4. The Spanish resident receives dividends from the shares of the Spanish firm they have just bought.
5. The Spanish resident now also buys shares of an Italian firm from a Turkish resident.
6. The Spanish resident receives dividends from the shares of the Italian firm they have just bought.
7. A Spanish resident moves permanently to Lisbon with a car.
8. A Spanish resident who holds British bonds moves to Rabat.
9. A Spanish resident sends money to their family abroad.
10. The Bank of Spain buys US Treasury bonds.
11. A Spanish resident buys a golden ring from a French jeweller.
12. The Bank of Spain buys gold from Japan.
13. A Spanish resident works for one month in a bar in Berlin.
14. A Danish tourist goes to a restaurant in Valencia.
15. A Spanish resident buys a house in Geneva.
16. A Spanish resident concedes a trade credit to a foreign firm to whom they have exported goods.
17. The Spanish government forgives part of the debt that Bolivia owes to the Spanish state.
18. A Chinese resident buys a patent from a Spanish firm.
19. An American billionaire buys Real Madrid.
20. A Spanish resident receives the Nobel prize.

2. We have studied the ”intertemporal approach to the current account” in the lectures. In this model, a representative agent maximizes utility over two periods:

$$\max_{C_1} u(C_1) + u(C_2), \tag{1}$$

subject to the following budget constraints:

$$z_0^{\text{HF}} + Y_1 = C_1 + z_1^{\text{HF}}, \tag{2}$$
$$z_1^{\text{HF}} + Y_2 = C_2 + z_2^{\text{HF}}. \tag{3}$$

The two-period intertemporal budget constraint is thus:

$$z_0^{\text{HF}} + Y_1 + Y_2 = C_1 + C_2 + z_2^{\text{HF}}. \tag{4}$$

The first-order condition for the problem is:

$$u'(C_1) + u'(C_2)(-1) = 0, \tag{5}$$

which gives rise to the ”Euler equation”:

$$u'(C_1) = u'(C_2). \tag{6}$$

Now assume logarithmic utility:

$$u(C) = \ln(C) \quad \Leftrightarrow \quad u'(C) = \frac{1}{C}. \tag{7}$$

Then we find that optimal consumption over the two periods is constant:

$$C_2 = C_1 = \frac{1}{2}(z_0^{\text{HF}} + Y_1 + Y_2 - z_2^{\text{HF}}), \tag{8}$$

and that the current accounts in both periods are given by the following equations:

$$CA_1 = Y_1 - C_1 = \frac{1}{2}(-z_0^{\text{HF}} + Y_1 - Y_2 + z_2^{\text{HF}}), \tag{9}$$
$$CA_2 = Y_2 - C_2 = \frac{1}{2}(-z_0^{\text{HF}} - Y_1 + Y_2 + z_2^{\text{HF}}). \tag{10}$$

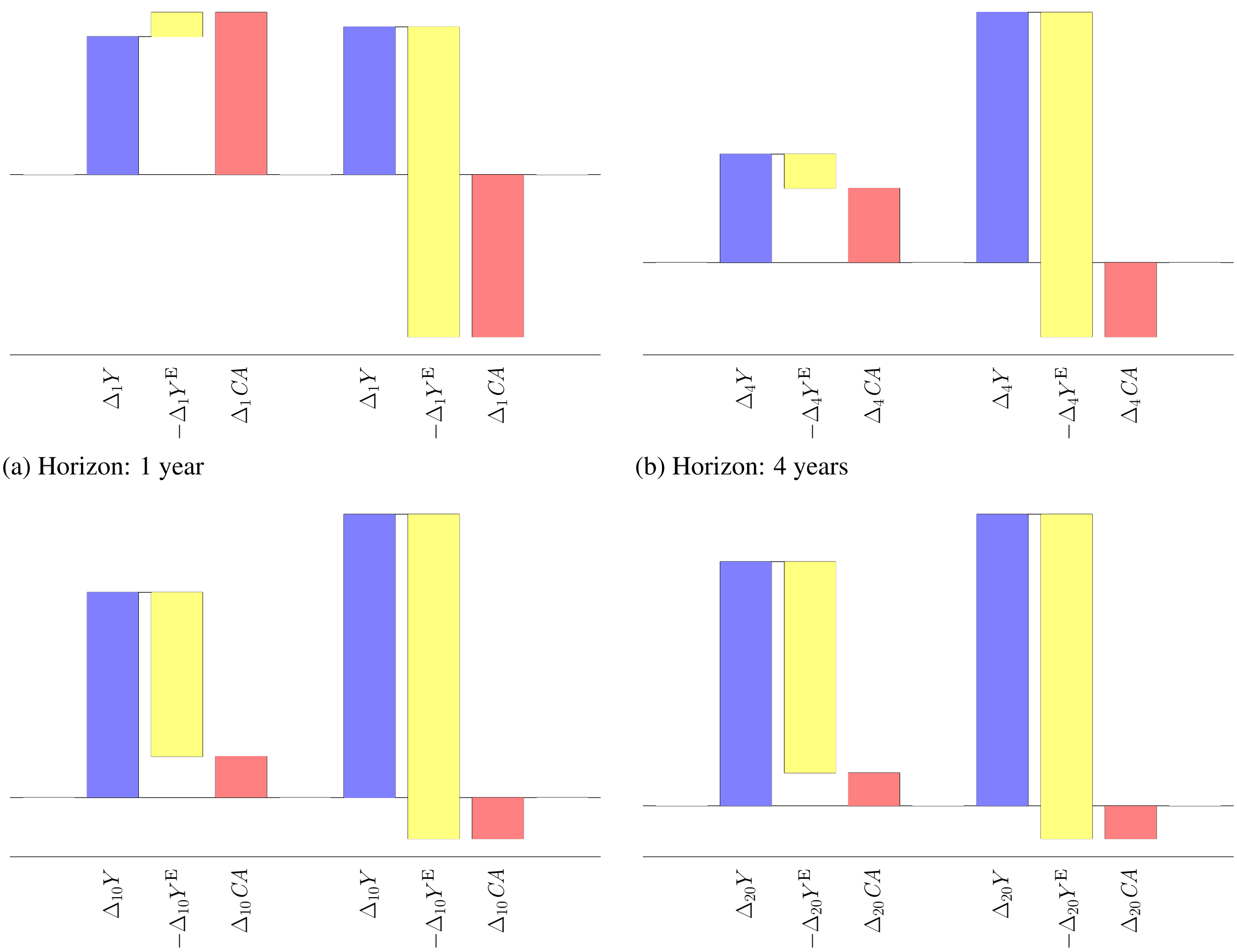
Now do the following:

(a) Derive the above solution of the model using the method of Lagrange multipliers.

(b) Develop the ”intertemporal approach to the current account” for three instead of two periods. We may interpret period 1 as the present, period 2 as the near future and period 3 as the distant future.

   i) What are C_1, C_2 and C_3 equal to in the solution of the three-period version of the model?

   ii) What are CA_1, CA_2 and CA_3 equal to in the solution of the three-period version of the model?



(a) Horizon: 1 year

(b) Horizon: 4 years

(c) Horizon: 10 years

(d) Horizon: 20 years

Figure 1: Empirical current account changes. Source: Müller-Plantenberg (2017).

iii) How does a temporary, positive income shock in period 1 affect the current accounts in the three periods? Explain your result.

iv) How does a permanent, positive income shock in periods 1, 2 and 3 affect the current accounts in the three periods? Explain your result.

v) What information do you need to compute the theoretical prediction of CA_1?

vi) What information do you need to compute the theoretical prediction of ΔCA_2 (= $CA_2 - CA_1$)?

vii) Suppose we want to find out whether the predictions of the ”intertemporal approach to the current account” regarding the effects of temporary and permanent income shocks are borne out by the data. In the light of your answers to the previous two questions, explain why it may be a good idea to use differenced, rather than non-differenced, data for this purpose.

viii) In the end, are the predictions of the ”intertemporal approach to the current account” regarding the effects of income shocks borne out by the data? To answer this question, you may take a look at figure 1.

(c) Now consider again the two-period version of the "intertemporal approach to the current account". However, let us now include investment in the model. The budget constraints for periods 1 and 2 are thus given by:

$$z_0^{\mathrm{HF}} + Y_1 + = C_1 + I_1 + z_1^{\mathrm{HF}}, \tag{11}$$
$$z_1^{\mathrm{HF}} + Y_2 + = C_2 + I_2 + z_2^{\mathrm{HF}}. \tag{12}$$

i) What form does the two-period intertemporal budget constraint take?

ii) What are the optimal values of C_1 and C_2?

iii) What values do CA_1 and CA_2 take in the solution of the model?

iv) If investment is included in the model, will the prediction of the "intertemporal approach to the current account" regarding the effect of temporary income shocks on the current account still hold? (Note that investment in a given period typically raises I in this period and Y in this and the following periods. For investment to make sense, the total rise in income should more than offset the cost of investment.)

(d) Consider again the two-period version of the "intertemporal approach to the current account", this time without investment. The maximization problem is as before, except that a "discount factor" β is added, which reduces the weight of $u(C_2)$ in the lifetime utility:

$$\max_{C_1} u(C_1) + \beta u(C_2). \tag{13}$$

The parameter β is fixed and normally assumed to be slightly smaller than one, say 0.95. The budget constraints for periods 1 and 2 are also modified and given by:

$$z_0^{\mathrm{HF}} + Y_1 + r z_0^{\mathrm{HF}} = C_1 + z_1^{\mathrm{HF}}, \tag{14}$$
$$z_1^{\mathrm{HF}} + Y_2 + r z_1^{\mathrm{HF}} = C_2 + z_2^{\mathrm{HF}}. \tag{15}$$

i) Show that the two budget constraints for periods 1 and 2 represent balance of payments identities?

ii) Explain what it means when it is said that the discount factor β measures how "impatient" the representative agent is.

iii) What do the terms $r z_0^{\mathrm{HF}}$ and $r z_1^{\mathrm{HF}}$ represent?

iv) What form does the two-period intertemporal budget constraint take?

v) What are the optimal values of C_1 and C_2?

vi) What values do CA_1 and CA_2 take in the solution of the model?

vii) If we include investment spending, I_1 and I_2, in the model, what form does the two-period intertemporal budget constraint take and what are the values of C_1, C_2, CA_1 and CA_2 in the solution of the model?

3. Suppose you want to invest your wealth W (with $W = 10$) and can choose between three assets A, B and C. The returns of the three assets (R^A, R^B, R^C) take the following values with equal probability: (2, 8, 8), (2, 6, 10), (2, 8, 2), (2, 6, 16).

   Using the model of international investment that we studied in class and assuming that the coefficient of risk aversion, λ, is equal to one, answer the following questions:

   (a) Draw a scatter plot with the returns of asset C on the vertical axis and those of B on the horizontal axis. Are the returns positively or negatively correlated?

   (b) Compute the average returns of the three assets.

   (c) Compute the deviations from the average returns of the three assets.

   (d) Compute the variances of the returns of the three assets.

   (e) Compute the standard deviations of the returns of the three assets.

   (f) Compute the covariances between assets A and B, assets A and C and assets B and C.

   (g) Compute the correlations between assets A and B, assets A and C and assets B and C.

   (h) What are the optimal portfolio shares w^A, w^B and w^C for the three assets.

   (i) Is it optimal to go short in any of the assets? If so, why?

   (j) What are the returns of the optimal portfolio in the four states of the world. What is the average return?

   (k) Show that the variance of the returns of the optimal portfolio is lower than the variances of assets B and C? Comment briefly on why it is lower.

4. The following questions are based on the table ”A feast of burgernomics: The Big Mac index”, which was published by the news magazine *The Economist* in 2007.
   (a) Define the real exchange rate.
   (b) Why does the *Economist* update and publish the Big Mac index every year? What are the advantages and disadvantages of the Big Mac index?
   (c) Compare the price of the Big Mac in Africa, Asia, Australia, Europe, Latin America and North America.
      i) In which continents is the Big Mac most expensive and in which continents is it cheapest?
      ii) How do the countries with the highest prices differ from the countries with the lowest prices?
   (d) i) Compare the dollar price of the Big Mac in the United States with the dollar price of the Big Mac in Argentina.
      ii) Now repeat the comparison, but this time with prices expressed in yen. Is the result the same?
   (e) i) Calculate the real exchange rate of the United States vis-à-vis South Africa. Also take the natural logarithm and the logarithm to the base 2 of the real exchange rate.
      ii) Now calculate the real exchange rate of South Africa vis-à-vis the United States. Also take the natural logarithm and the logarithm to the base 2 of the real exchange rate.
      iii) Compare the results.
   (f) Using the prices of the Big Mac, calculate the highest and the lowest real exchange rate that existed in the world in 2007. Which countries are we talking about and what are the highest and the lowest real exchange rates? Express those real exchange rates also in terms of natural logarithms and logarithms to the base 2.
   (g) i) Suppose that purchasing power parity holds between China and the United States. What should the nominal exchange rate between the Chinese yuan and the US dollar be? To what extent was the nominal exchange rate of the yuan vis-à-vis the US dollar on 31 January 2007 over- or undervalued?
      ii) Now suppose that the real exchange rate of China vis-à-vis the United States tends to equal to 0.5 in the medium and long run. Taking this into account, what level of the nominal exchange rate of the yuan vis-à-vis the US dollar would be adequate (= compatible with the mentioned real exchange rate)? Under the new assumption on the real exchange rate, to what extent was the nominal exchange rate of the yuan vis-à-vis the US dollar on 31 January 2007 over- or undervalued?

5. Use the Lagrange method to find the equilibrium values of the endogenous variables and Lagrange multipliers in the following constrained optimization problems. C^{HH} and C^{HF} stand for consumption by domestic agents of domestic and foreign goods, respectively. The variable x^{HX} stands for the domestic currency (:HC) that the home agent exchanges for the foreign currency (:FC) in the foreign exchange market.

   (a) [Purchasing power parity (PPP).] Suppose a domestic consumer wants to maximize their utility by consuming domestic and foreign goods:

$$\max_{C^{\text{HH}},C^{\text{HF}},x^{\text{HX}}} u(C^{\text{HH}} + C^{\text{HF}}), \tag{16}$$

   subject to:

$$\begin{aligned} P^{\text{H}}C^{\text{HH}} &= P^{\text{H}}Y^{\text{H}} - x^{\text{HX}}, \quad \text{LM: } \lambda^{:\text{HC}}, & (17)\\ P^{\text{F}}C^{\text{HF}} &= Sx^{\text{HX}}, \qquad\qquad \text{LM: } \lambda^{:\text{FC}}, & (18)\\ & & (19) \end{aligned}$$

   where Y^{H}, P^{H} and P^{F} are given.

   Apart from C^{HH}, C^{HF} and S and the Lagrange multipliers of the constraints, also calculate the real exchange rate Q, which is given by:

$$Q = \frac{SP^{\text{H}}}{P^{\text{F}}}. \tag{20}$$

   (b) [Imperfect substitution of domestic and foreign goods.] Suppose a domestic consumer wants to maximize their utility by consuming domestic and foreign goods:

$$\max_{C^{\text{HH}},C^{\text{HF}},x^{\text{HX}}} u(C^{\text{HH}}) + u(C^{\text{HF}}), \tag{21}$$

   subject to:

$$\begin{aligned} P^{\text{H}}C^{\text{HH}} &= P^{\text{H}}Y^{\text{H}} - x^{\text{HX}}, \quad \text{LM: } \lambda^{:\text{HC}}, & (22)\\ P^{\text{F}}C^{\text{HF}} &= Sx^{\text{HX}}, \qquad\qquad \text{LM: } \lambda^{:\text{FC}}, & (23) \end{aligned}$$

   where Y^{H}, P^{H} and P^{F} are given.

   Apart from C^{HH}, C^{HF} and S and the Lagrange multipliers of the constraints, also calculate the real exchange rate Q.

   (c) [Carry trades and uncovered interest parity (UIP).] Suppose an investor wants to maximize their return by investing in either domestic or foreign currency, or both:

$$\max_{M_2^{:\text{HC}},b_1^{:\text{HC}},b_1^{:\text{FC}},x_1^{\text{HX}},x_2^{\text{HX}}} M_2^{:\text{HC}}, \tag{24}$$

   subject to:

$$\begin{aligned} b_1^{:\text{HC}} &= M_0^{:\text{HC}} - x_1^{\text{HX}}, & (25)\\ b_1^{:\text{FC}} &= S_1 x_1^{\text{HX}}, & (26)\\ M_2^{:\text{HC}} &= (1 + R^{:\text{HC}})b_1^{:\text{HC}} - x_2^{\text{HX}}, & (27)\\ 0 &= (1 + R^{:\text{FC}})b_1^{:\text{FC}} + S_2 x_2^{\text{HX}}, & (28) \end{aligned}$$

   where $M_0^{:\text{HC}}$, $R^{:\text{FC}}$ and either $R^{:\text{HC}}$ or S_1 are given.

References

Müller-Plantenberg, Nikolas A. Accounting for current account changes: What matters is spending, not income. *Cuadernos Económicos de ICE*, vol. 1, no. 94, Dec. 2017, 11–31.