Appendices

S1 Solution details

This section shows how to solve the problem of the representative consumer-investor in equations 2 and 5. The first one to solve this kind of problem was Merton (1971). A particularly intuitive way to solve the problem, however, is the method of symmetry (see Chang, 2004). The idea behind this method is that it is sometimes possible to carry out a change of variables that affects the objective function but leaves the law of motion of the problem invariant. Such a transformation is called a symmetry since the dynamics of the model is left unchanged. If the transformed objective function can be related to the objective function of the original problem, then it is possible to ascertain the functional form of the value function.

In the present context, the objective function is given by equation 2 and the law of motion governing the dynamics of the model by the controlled diffusion process in equation 5. Suppose wealth is doubled at all times, both at home and abroad. Intuition tells us that consumption by the domestic and foreign consumer-investors should be doubled at all times, too. If we further assume that the shares of all assets remain unchanged, then we are in effect considering the following transformation:

$$\Psi(a_0^{\mathrm{H}}, a_0^{\mathrm{F}}, c_1^{\mathrm{H}}, c_1^{\mathrm{F}}, w_i) = (\psi a_0^{\mathrm{H}}, \psi a_0^{\mathrm{F}}, \psi c_1^{\mathrm{H}}, \psi c_1^{\mathrm{F}}, w_i), \tag{32}$$

where $i \in \{e^{\mathrm{HH}}, e^{\mathrm{FF}}, e^{\mathrm{HF}}, e^{\mathrm{FH}}, b^{\mathrm{HF}}, b^{\mathrm{FH}}\}$ and w_i is defined as in section 4.2.

This leads to the following transformed financial constraint of the domestic consumer-investor:

$$
\begin{aligned}
\psi a_1^{\mathrm{H}} dt = & \left(\sum_i (\pi_i - r) w_i \psi a_0^{\mathrm{H}} - \sum_j (\pi_j - r) w_j \psi a_0^{\mathrm{F}} + r \psi a_0^{\mathrm{H}} - \psi c_1^{\mathrm{H}} \right) dt \\
& + \sum_i w_i \psi a_0^{\mathrm{H}} \zeta_i d\omega_i - \sum_j w_j \psi a_0^{\mathrm{F}} \zeta_j d\omega_j.
\end{aligned} \tag{33}
$$

where $i \in \{e^{\mathrm{HH}}, e^{\mathrm{HF}}, b^{\mathrm{HF}}\}$, $j \in \{e^{\mathrm{FH}}, b^{\mathrm{TH}}\}$. The transformation described in equation 32 is a symmetry since the transformed law of motion in equation 33 describes the same dynamics as the initial law of motion in equation 5 (and the same for the foreign-country counterparts of the respective equations).

Using this symmetry, the transformed objective function is:

$$
\mathrm{E}\left(\int_0^{\infty} e^{-\rho t} \frac{(\psi c_{1,t}^{\mathrm{H}})^{1-\gamma}}{1-\gamma} \, dt \right) = \psi^{1-\gamma} \, \mathrm{E}\left(\int_0^{\infty} e^{-\rho t} \frac{(c_{1,t}^{\mathrm{H}})^{1-\gamma}}{1-\gamma} \, dt \right). \tag{34}
$$

It follows that:

$$
V(\psi a_0^{\mathrm{H}}) = \psi^{1-\gamma} V(a_0^{\mathrm{H}}). \tag{35}
$$

Setting $\psi = 1/a_0^{\mathrm{H}}$ and using the optimality condition in equation 7, this leads to:

$$
V(a_0^{\mathrm{H}}) = V(1)(a_0^{\mathrm{H}})^{1-\gamma} = \frac{A^{-\gamma}}{1-\gamma}(a_0^{\mathrm{H}})^{1-\gamma}. \tag{36}
$$

From the Bellman equation 6, one can now compute A, the share of wealth that is consumed:

$$
A = \frac{\rho}{\gamma} - \frac{1-\gamma}{\gamma} \left[r + \frac{1}{2} \sum_{i=1}^{n-1} \frac{(\mu_i - r)^2}{\gamma \sigma_i^2} \right]. \tag{37}
$$

In the case of logarithmic utility, similar reasoning leads to the following value function:

$$
V(a_0^{\mathrm{H}}) = \frac{1}{A} \ln(a_0^{\mathrm{H}}) + V(1), \tag{38}
$$

where $A = \rho$ and $V(1)$ can be calculated using the Bellman equation.

S2 Real investment dynamics

When asset returns and volatilities are held constant, the stock of domestic equity, $e_0^{\mathrm{TH}} = e_0^{\mathrm{HH}} + e_0^{\mathrm{FH}}$, evolves according to the differential equation

$$e_1^{\mathrm{H}} = \hat{a}_0^{\mathrm{H}} e_0^{\mathrm{H}}, \tag{39}$$

whose solution is:

$$e_{0,t}^{\mathrm{H}} = C_e e^{\hat{a}_0^{\mathrm{H}} t}. \tag{40}$$

The solution of the differential equation of the domestic capital stock, equation 12, yields:

$$\begin{aligned} k_{0,t}^{\mathrm{H}} &= e^{(\hat{a}_0^{\mathrm{H}} - \lambda')t} \left(C_k + \int \lambda' C_e e^{\hat{a}_0^{\mathrm{H}} t} e^{-(\hat{a}_0^{\mathrm{H}} - \lambda')t} dt \right) \\ &= C_k e^{(\hat{a}_0^{\mathrm{H}} - \lambda')t} + C_e e^{\hat{a}_0^{\mathrm{H}} t}, \end{aligned} \tag{41}$$

where $\lambda' = \lambda(1 + \hat{a}_0^{\mathrm{H}})$.

Let Δ_t be the gap between the stock market value and the actual value of the domestic capital stock; that is:

$$\Delta_t = e_{0,t}^{\mathrm{H}} - k_{0,t}^{\mathrm{H}} = -C_k e^{(\hat{a}_0^{\mathrm{H}} - \lambda')t}, \tag{42}$$

where $C_k = -(e_0^{\mathrm{H}} - k_0^{\mathrm{H}})$. Suppose moreover that a percentage χ of a given stock market overvaluation disappears after l years (provided asset returns and volatilities are held constant). This then implies:

$$(1 - \chi)\Delta_0 = \Delta_0 e^{(\hat{a}_0^{\mathrm{H}} - \lambda')l} \quad \Leftrightarrow \quad \lambda = \frac{\hat{a}_0^{\mathrm{H}} - \frac{\ln(1-\chi)}{l}}{1 + \hat{a}_0^{\mathrm{H}}}. \tag{43}$$

In section 4, we assume that $\hat{a}_0^{\mathrm{H}} = 0.03$, $\chi = 0.50$ and $l = 5$, which implies a value of λ of 0.1637. As it turns out, however, the qualitative results are robust with respect to the choice of χ, so that choosing a higher or lower value for this parameter (say, between 0.1 and 0.9) would not alter the economic outcome in a qualitatively significant way.

S3 Approximation of currency market pressure

This appendix shows how the approximation in equations 22 and 24 is obtained. The derivation of currency market pressure, $\tilde{m}_0$, in equation 24 is based on the following calculation:

$$\begin{aligned}
\tilde{m}_{0,t} &= m_{0,t}^{\text{H:FC}} - m_{0,t}^{\text{F:HC}} \\
&= S_{0,t}^{-(1-\upsilon)} \int_{-\infty}^{t} S_{0,\tau}^{1-\upsilon} \left(\phi\, x_{1,\tau}^{\text{H}} + b_{1,\tau}^{\bar{\text{F}}\text{H}}\right) d\tau \\
&\quad - S_{0,t}^{\upsilon} \int_{-\infty}^{t} S_{0,\tau}^{-\upsilon} \left(-(1-\phi)\, x_{1,\tau}^{\text{H}} + b_{1,\tau}^{\bar{\text{H}}\text{F}}\right) d\tau \\
&\approx S_{0,t}^{-(1-\upsilon)} \int_{-\infty}^{t} \phi x_{1,\tau}^{\text{H}} + b_{1,\tau}^{\bar{\text{F}}\text{H}} d\tau \\
&\quad - S_{0,t}^{\upsilon} \int_{-\infty}^{t} -(1-\phi) x_{1,\tau}^{\text{H}} + b_{1,\tau}^{\bar{\text{H}}\text{F}} d\tau \\
&= S_{0,t}^{-(1-\upsilon)} \left(\phi x_{0,t}^{\text{H}} + b_{0,t}^{\bar{\text{F}}\text{H}}\right) - S_{0,t}^{\upsilon} \left(-(1-\phi) x_{0,t}^{\text{H}} + b_{0,t}^{\bar{\text{H}}\text{F}}\right) \\
&\approx x_{0,t}^{\text{H}} - b_{0,t}^{\bar{\text{H}}\text{F}} + b_{0,t}^{\bar{\text{F}}\text{H}}.
\end{aligned} \tag{44}$$

The first approximation is based on a first-order Taylor series expansion of the integrands around $S_{0,\tau} = 1$, $b_{1,\tau}^{\bar{\text{H}}\text{F}} = 0$, $b_{1,\tau}^{\bar{\text{F}}\text{H}} = 0$ and $x_{1,\tau}^{\text{H}} = 0$, the second one on a first-order Taylor series expansion of the whole expression around $S_{0,t} = 1$, $b_{0,t}^{\bar{\text{H}}\text{F}} = 0$, $b_{0,t}^{\bar{\text{F}}\text{H}} = 0$ and $x_{0,t}^{\text{H}} = 0$.