International macroeconomics (intermediate level)

Lecture notes

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1 Balance of payments

1.1 The structure of the balance of payments

1.1.1 Basic structure

1. Current Account
2. Capital account
3. Financial account

1.1.2 Detailed structure (1)

1. Current Account
   (a) Goods
   (b) Services
   (c) Income
   (d) Current transfers
2. Capital account
   (a) Capital transfers
   (b) Acquisition and disposal of non-produced, nonfinancial assets
3. Financial account
   (a) Direct investment
   (b) Portfolio investment
   (c) Other investment
   (d) Reserves

1.1.3 Detailed structure (2)

1. Current Account
   (a) Goods
   (b) Services
      i. Transportation
      ii. Travel
      iii. Communications services
      iv. Construction services
      v. Insurance services
      vi. Financial services
      vii. Computer and information services
      viii. Royalties and license fees
      ix. Other business services
      x. Personal, cultural, and recreational services
      xi. Government services

   (c) Income
      i. Compensation of employees
      ii. Investment income
   (d) Current transfers
      i. General government
      ii. Other sectors
         A. Workers' remittances
         B. Other transfers

2. Capital account
   (a) Capital transfers
      i. Debt forgiveness
      ii. Migrants' transfers
      iii. Other
   (b) Acquisition and disposal of non-produced, nonfinancial assets

3. Financial account
   (a) Direct investment
       i. Equity capital
       ii. Reinvested earnings
       iii. Other capital
   (b) Portfolio investment
       i. Equity securities
       ii. Debt securities
   (c) Other investment
       i. Trade credits
       ii. Loans
       iii. Currency and deposits
       iv. Other assets
   (d) Reserves

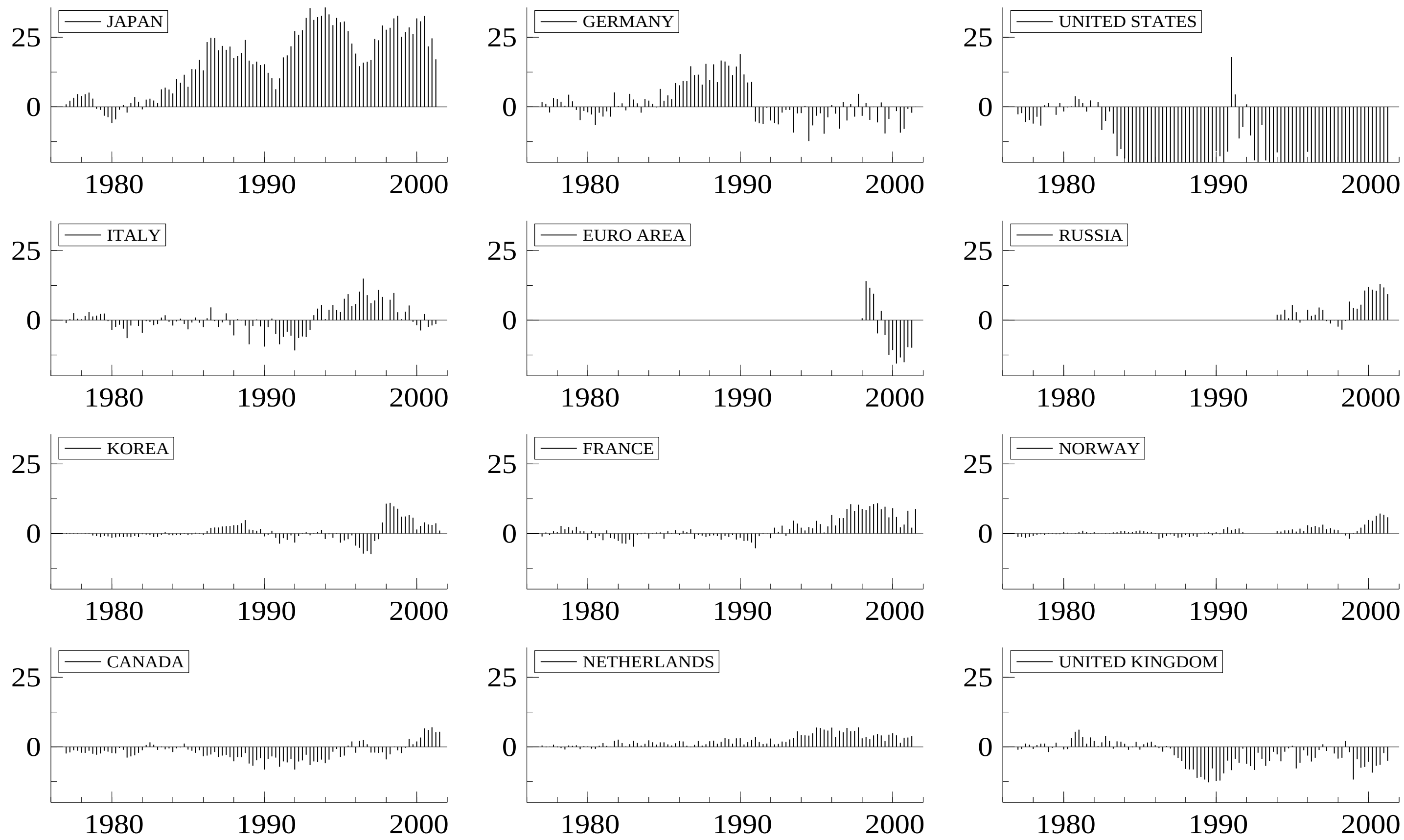
1.2 Credits and debits

Credits:

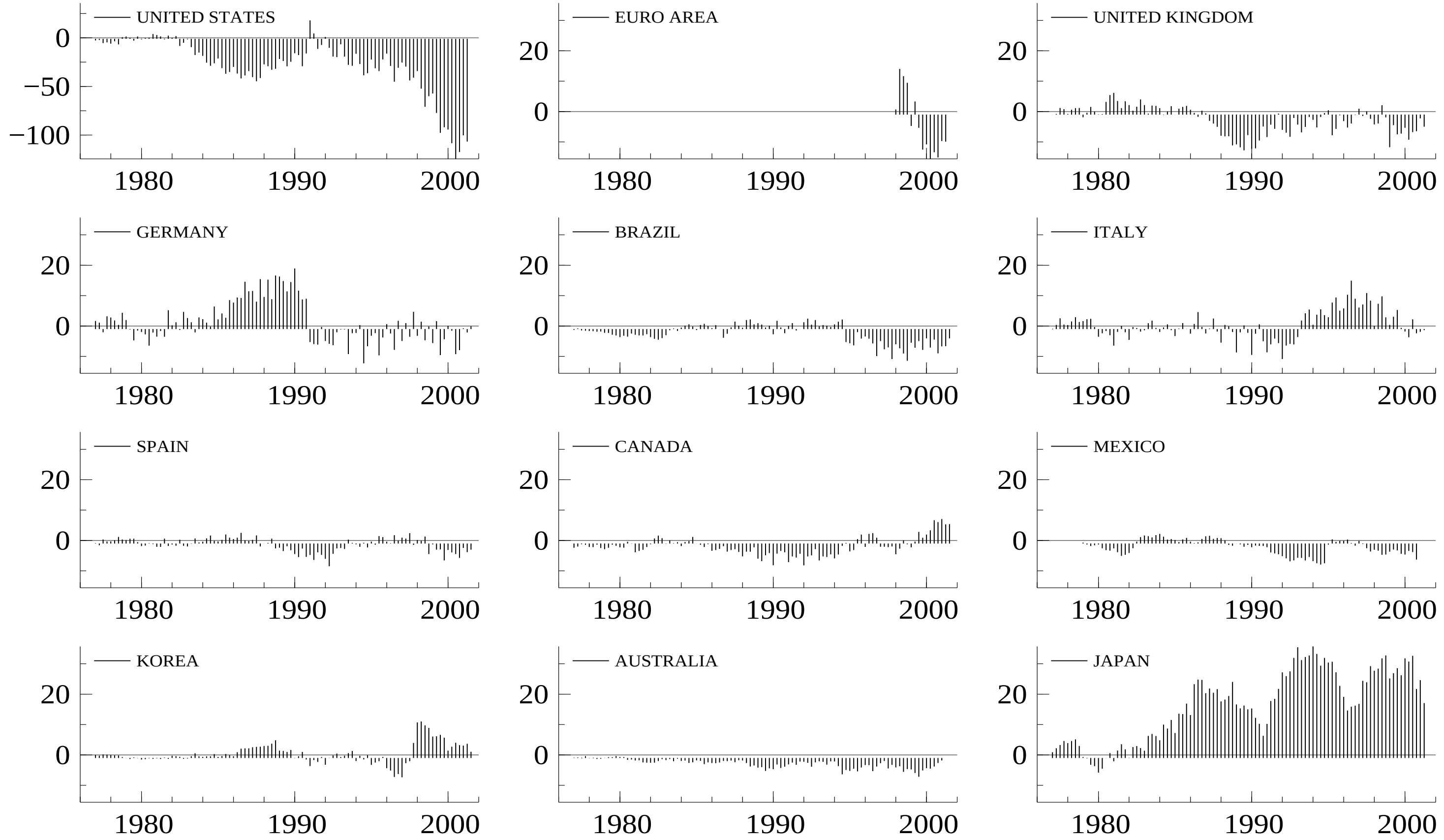
1. Current account
    - Exports of goods and services
    - Income received
    - Current transfers received
2. Capital account
    - Capital transfers received
    - Acquisition of non-produced, non-financial assets
3. Financial account
    - Decreases in financial assets
    - Increases in liabilities

Debits:

1. Current account
   - Imports of goods and services
   - Income paid
   - Current transfers sent
2. Capital account
   - Capital transfers sent
   - Disposal of non-produced, non-financial assets
3. Financial account
   - Increases in financial assets
   - Decreases in liabilities



Large current account surpluses.



Large current account deficits.

1.3 National and international accounts

1.3.1 National and international accounting

Important variables:

- GNP: gross national product
- GDP: gross domestic product
- GNI: gross national income
- GNDI: gross national disposable income
- GNE: gross national expenditure

1.3.2 Change in the usage of the terms GDP and GNP

	Traditional definition	System of National Accounts (SNA 1993)
Gross domestic product (GDP)	Production within a **country's borders**	Production by a **country's residents**
Gross national product (GNP)	Production by a **country's residents**	—

1.3.3 Important relationships

$$GDP = GNE + TB, \tag{1}$$
$$GNI = GDP + NFIA, \tag{2}$$
$$GNDI = C + I + G + CA = GNI + NUT, \tag{3}$$
$$GNE = C + I + G = GNDI - CA = GNDI + FA + KA, \tag{4}$$

where

$$\begin{aligned}
TB &= EX_{GS} - IM_{GS} = \text{trade balance (or commercial balance)}\\
&= \text{goods balance} + \text{services balance},\\
NFIA &= EX_{FS} - IM_{FS} = \text{net factor income from abroad},\\
NUT &= UT_{\text{received from abroad}} - UT_{\text{sent abroad}} = \text{net unilateral transfers},\\
CA &= \text{current account},\\
FA &= EX_{A} - IM_{A} = \text{financial account},\\
KA &= AT_{\text{received from abroad}} - AT_{\text{sent abroad}} = \text{capital account}
\end{aligned}$$

and *EX* stands for exports, *IM* for imports, *GS* for goods and services, *FS* for factor services, *UT* for unilateral transfers, *A* for assets and *AT* for asset transfers.

Note:

$$\begin{aligned} CA &= TB + NFIA + NUT \\ &= -FA - KA \\ &= GNDI - GNE. \end{aligned} \tag{5}$$

1.3.4 Notation

- H: domestic resident
- F: foreign resident
- Superscript H: in home country
- Superscript F: in foreign country
- A $\Rightarrow$ B: A sells goods and services to B
- A $\leftarrow$ B: A receives factor income from B
- A $\curvearrowleft$ B: A receives unilateral transfer from B
- A $\dashleftarrow$ B: A receives asset transfer from B

	GNE	TB	GDP	NFIA	GNI	NUT	GNDI	CA	KA	FA	GDP (border-based)	GDP (border-based) - GDP
	1	2	3	4	5	6	7	8	9	10	11	12
	7+9+10		1+2		3+4		5+6	2+4+6		-(8+9)		11-3
$H^H \Rightarrow H^H$	+		+		+		+				+	
$H^H \Rightarrow H^F$	+		+		+		+				+	
$H^H \Rightarrow F^H$		+	+		+		+	+		-	+	
$H^H \Rightarrow F^F$		+	+		+		+	+		-	+	
$H^F \Rightarrow H^H$	+		+		+		+					-
$H^F \Rightarrow H^F$	+		+		+		+					-
$H^F \Rightarrow F^H$		+	+		+		+	+		-		-
$H^F \Rightarrow F^F$		+	+		+		+	+		-		-
$F^H \Rightarrow H^H$	+	-						-		+	+	+
$F^H \Rightarrow H^F$	+	-						-		+	+	+
$F^H \Rightarrow F^H$											+	+
$F^H \Rightarrow F^F$											+	+
$F^F \Rightarrow H^H$	+	-						-		+		
$F^F \Rightarrow H^F$	+	-						-		+		
$F^F \Rightarrow F^H$												
$F^F \Rightarrow F^F$												
$H \leftarrow F$				+	+		+	+		-		
$F \leftarrow H$				-	-		-	-		+		
$H \curvearrowleft F$						+	+	+		-		
$F \curvearrowleft H$						-	-	-		+		
$H \dashleftarrow F$									+	-		
$F \dashleftarrow H$									-	+		

1.3.5 The United Nations' System of National Accounts (SNA)

GDP, GNI and GNDI according to the System of National Accounts 2008:

- 2.138 Basically, **GDP** derives from the concept of value added. Gross value added is the difference between output and intermediate consumption. GDP is the **sum of gross value added of all resident producer units** plus that part (possibly the total) of taxes on products, less subsidies on products, that is not included in the valuation of output.
- 2.143 [...] **GNI** is equal to GDP less primary incomes payable to non-resident units plus primary incomes receivable from non-resident units. In other words, GNI is equal to **GDP less** taxes (less subsidies) on production and imports, **compensation of employees and property income payable to the rest of the world plus the corresponding items receivable from the rest of the world.** Thus GNI is the sum of gross primary incomes receivable by resident institutional units or sectors. In contrast to GDP, GNI is not a concept of value added, but a concept of income.

- 2.145 [...] **Gross national disposable income** is equal to **GNI less current transfers** (other than taxes, less subsidies, on production and imports) **payable to non-resident units, plus the corresponding transfers receivable by resident units from the rest of the world.** Gross national disposable income measures the income available to the total economy for final consumption and gross saving. [...] National disposable income is the sum of disposable income of all resident institutional units or sectors.
- 4.2 An **institutional unit** is an **economic entity that is capable, in its own right, of owning assets, incurring liabilities and engaging in economic activities and in transactions with other entities.** [...]
- 4.10 The **residence** of each institutional unit is the economic territory with which it has the strongest connection, in other words, its **centre of predominant economic interest.** The concept of economic territory in the SNA coincides with that of the BPM6. [...]
- 4.14 An institutional unit has a **centre of predominant economic interest** in an economic territory when there exists, **within the economic territory,** some location, dwelling, place of production, or other premises on which or from which the unit engages and intends to continue engaging, either indefinitely or over a finite but long period of time, in **economic activities and transactions** on a significant scale. [...] Actual or intended location for **one year or more** is used as an operational definition [...]

- 4.15 The **concept of residence** in the SNA is exactly the same as in BPM6. Some key consequences follow:
  - a. The **residence of individual persons** is **determined by that of the household** of which they form part and **not by their place of work.** All members of the same household have the same residence as the household itself, even though they may cross borders to work or otherwise spend periods of time abroad. If they work and reside abroad so long that they acquire a centre of economic interest abroad, they cease to be members of their original households;
  - b. **Unincorporated enterprises** that are not quasi-corporations are not separate institutional units from their owners and, therefore, have the **same residence as their owners;**

- c. **Corporations and NPIs** [non-profit institutions] may **normally** be expected to have a **centre of economic interest in the country in which they are legally constituted and registered. Corporations may be resident in countries different from their shareholders** and **subsidiary corporations may be resident in countries different from their parent corporations. When a corporation,** or unincorporated enterprise, **maintains a branch, office or production site in another country in order to engage in production over a long period of time** (usually taken to be one year or more) but without creating a subsidiary corporation for the purpose, **the branch, office or site is considered to be a quasi-corporation** (that is, a separate institutional unit) **resident in the country in which it is located;**
- d. **Owners of land, buildings and immovable structures in the economic territory of a country,** or units holding long leases on either, are deemed **always** to **have a centre of economic interest in that country,** even if they do not engage in other economic activities or transactions in the country. **All land and buildings are therefore owned by residents;**
- e. **Extraction of subsoil resources can only be undertaken by resident institutional units.** An enterprise that will undertake extraction is deemed to become resident when the requisite licences or leases are issued, if not before;

...

- 4.23 The **total economy** is defined as the **entire set of resident institutional units.** [...]
- 4.24 All **resident institutional units** are **allocated to** one and only one of the following **five institutional sectors:**
  - The **non-financial corporations** sector;
  - The **financial corporations** sector;
  - The **general government** sector;
  - The **non-profit institutions serving households** sector;
  - The **households** sector.

1.3.6 National savings and the current account

National savings in a **closed economy:**

$$S = Y - C - G \qquad \Leftrightarrow \qquad S = I. \tag{6}$$

Saving only by capital accumulation.

National savings in an **open economy:**

$$S = Y - C - G \qquad \Leftrightarrow \qquad S = I + CA. \tag{7}$$

Saving by capital accumulation or by the acquisition of external wealth.

1.3.7 Private and public savings

$$S^p = Y - T - C, \tag{8}$$

$$S^g = T - G, \tag{9}$$

where

$S^p :=$ private savings (disposable income which is not consumed),

$S^g :=$ public savings,

$T :=$ net taxes.

National savings:

$$S = S^p + S^g = (Y - T - C) + (T - G) = Y - C - G. \tag{10}$$

National income identity:

$$S^p = I + CA - S^g = I + CA + \underbrace{(G - T)}_{\text{budget deficit}}. \tag{11}$$

Therefore there are three possible uses of the national savings:

- investment in capital at home (I),
- acquisition of wealth abroad (CA),
- purchase of new government debt ($G - T$).

European Union (percentage of GDP):

Year	S^p	I	CA	$G-T$
1995	25.9	19.9	0.6	5.4
1996	24.6	19.3	1.0	4.3
1997	23.4	19.4	1.5	2.5
1998	22.6	20.0	1.0	1.6
1999	21.8	20.8	0.2	0.8

- Introduction of the euro in 1999
- Condition to participate: $(G-T)/Y < 3\%$
- Public deficit reduction compensated by the fall in private savings
- Possible explanations: Ricardian equivalence, boom in European financial assets

1.4 ”Imbalances” in the balance of payments

The term ”balance of payments” has two different meanings:

- the sum of the current account, capital account and financial account - always equals to zero
- changes in the official reserves (one of the elements in the balance of payments) - in general different to zero

Example of the second use: ”The country’s balance of payments shows a deficit.”

There are two cases in which one could refer to an ”imbalance” in the balance of payments:

- The balance of payments (changes in the official reserves) is in deficit.
- The current account is in deficit.

The first one can be the consequence of the second one.

1.4.1 Is running a current account deficit necessarily bad?

Many people think so (parallel: negative balance of the current account at a bank).

Recall:

- The current accounts of all the countries in the world always sum to zero.
- The current account expands a country's investment opportunities:
  - Closed Economy: $I = S$.
  - Open Economy: $I = S - CA$.
- A current account deficit can be financed by:
  - money receipts,
  - foreign financial investment,
  - external debt,
  - sale of private and public assets,
  - sale of official reserves.

1.4.2 Financing problems

External imbalances arise when the volume of national saving does not match the volume of national investment.

- Imbalances between saving and investment:

$$CA = -KA - FA = S - I \neq 0. \tag{12}$$

External financing problems arise if:

- the current account deteriorates or
- there is an outflow of assets or an increase in external liabilities.

Broadly speaking, there are two kinds of remedies:

- apply measures that push up the current account or
- apply measures that lead to an inflow of financial assets or a reduction in external liabilities.

Causes of the fall of net exports ($CA \downarrow$):

$$CA = X - M = S - I = (Y - C - G) - I. \tag{13}$$

- Fall of the prices of exports
- Restrictions to exports to foreign countries (for example tariffs)
- Fall in the demand of exports
- Rise in prices of imports (for example oil)
- Rise in demand of imports (for example consumption or investment boom)
- Decrease of the savings rate

Financial account:

- Capital flight to other countries
- Debt
  - High level (relative to net exports)
  - Debt of short maturity
  - Fall of net exports
- Shortage of official reserves

Measures to obtain that $CA\uparrow$:

$$Y = C + I + G + CA \qquad \Leftrightarrow \qquad CA = Y - C - I - G \tag{14}$$

- $Y\uparrow$
  - Measures to foster economic growth (difficult to achieve in the short term)
  - Balance of payments crises are often followed by falls in national income
- $C\downarrow$, $I\downarrow$ - private decisions
  - Price rises (wheat, fuel etc.)
  - Devaluation of the exchange rate
  - Reduction of private sector wages
  - Withdrawal from investment projects
- $G\downarrow$:
  - Reduction of public sector wages
  - Reduction of social security benefits
  - Withdrawal from investment projects

- Foreign aid
  - Development aid

Prevention of financing problems:

- Capital controls
- Debt
  - Level restrictions (relative to net exports)
  - Avoid debt with short maturity
  - Diversification of exports and imports, long-run contracts, oil reserves etc.
- Increase of official reserves
- Loans from the International Monetary Fund (IMF)

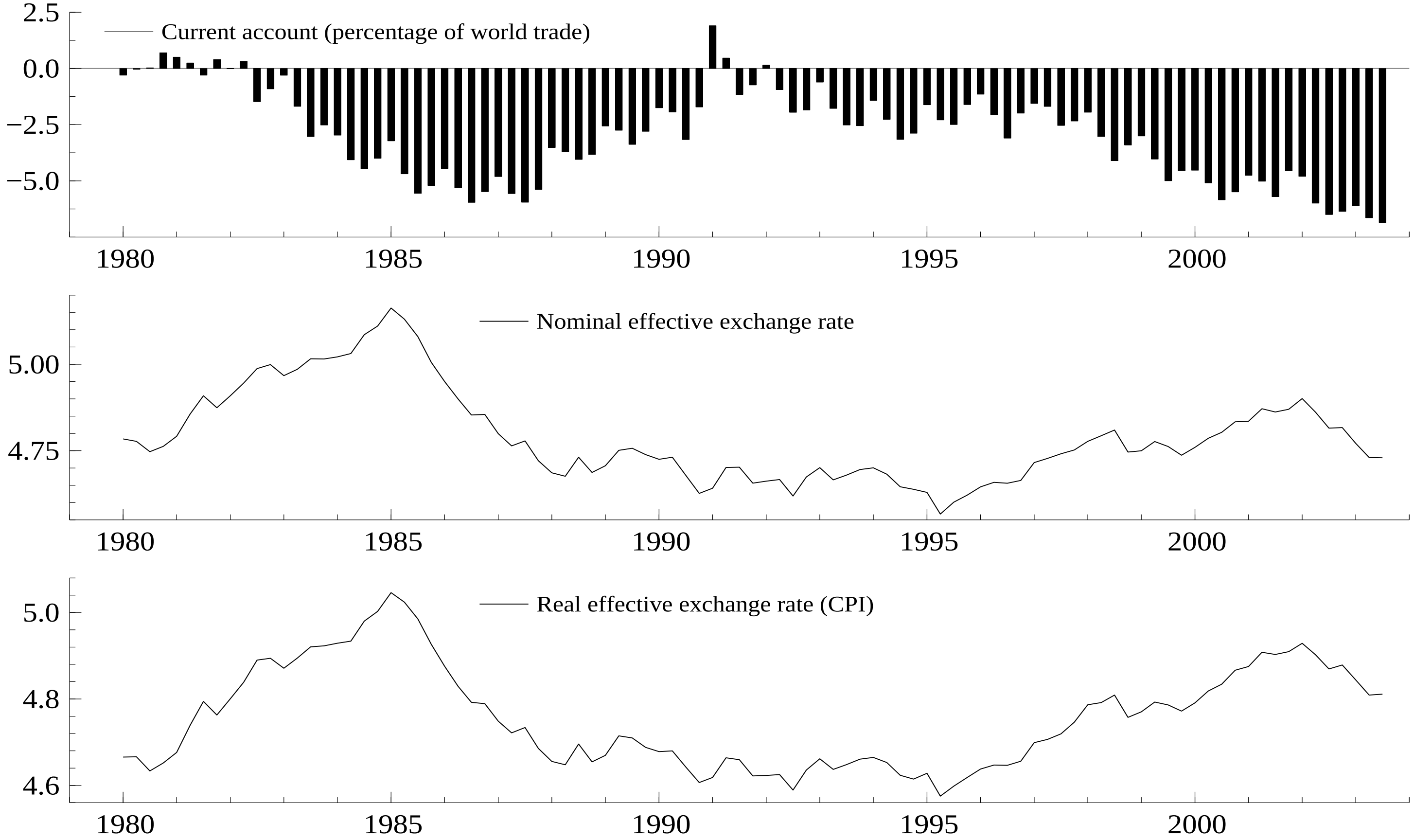
1.4.3 Exchange rates

Currency crisis (= sudden falls of the value of countries' currencies) related to current account deficits (examples):

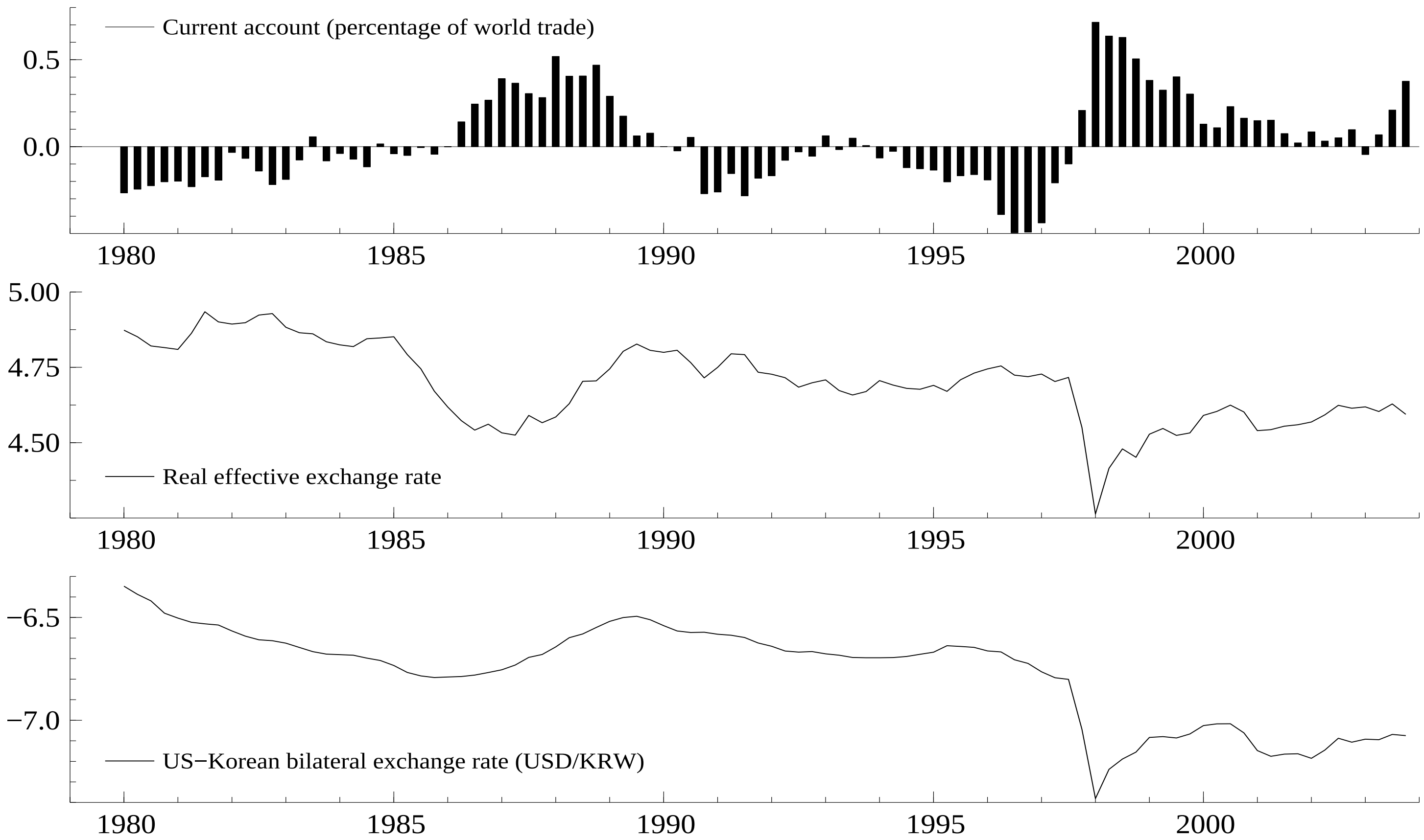
- United Kingdom (1992)
- Brasil (1999)
- Italy (1992)
- Spain (1993)
- Mexico (1994–1995)
- Korea (1997)

Continued depreciations:

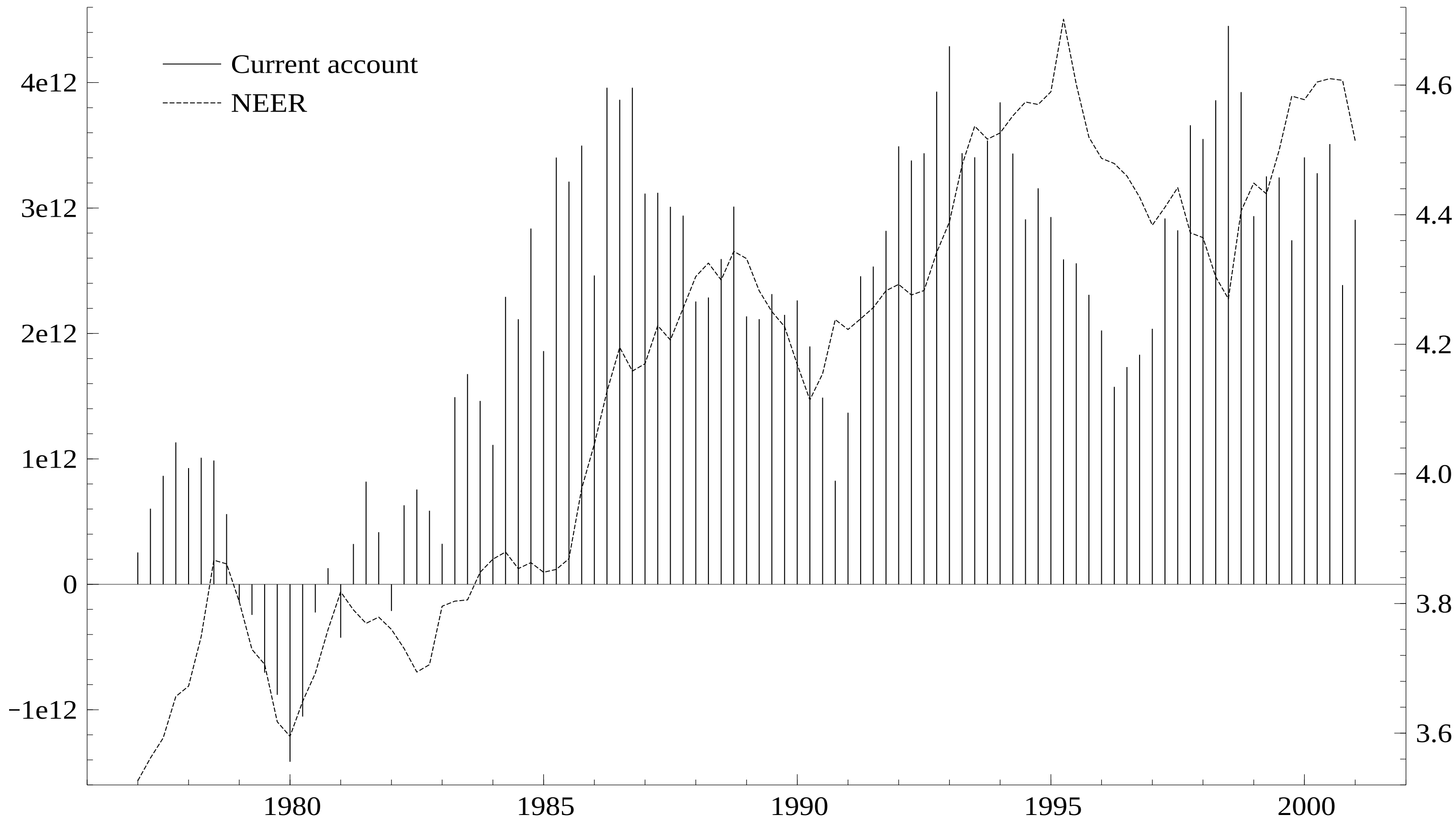
- United States (1985–1987, since 2002)
- Japan (1979)



US current account and exchange rate.



Korea's current account and exchange rate.



Japanese current account and exchange rate (1980s and 1990s).

1.4.4 Stages of development

- Poor countries have less capital (per capita) than rich countries.
- For a start, the return on capital should be higher in poor countries than in rich countries.
- Lending capital (= allowing a current account deficit) is a development strategy.
- Example of South Korea:
  - GDP per capita: 100$ in 1963, more than 20,000$ in 2005.
  - In 2006, South Korea was one of the richest economies in the world (as regards GDP, it was number 10 in nominal terms and number 14 after correcting for differences in price levels).
  - During the 1970s, South Korea had a current account deficit of more than 5% of GDP (on average).

Possible evolution of the balance of payments during a country's development:

- Poor countries invest strongly $\Rightarrow$ deficit in both the commercial balance and the balance on current account
- When the investments start to be profitable, exports increase, but the country has to pay the interests of the accumulated debt $\Rightarrow$ commercial balance surplus, current account deficit
- Gradually, the country reduces its debt and has less interest to pay $\Rightarrow$ current account surplus
- Eventually, the country reduces its liabilities and increases its assets $\Rightarrow$ the country becomes a net creditor
- At a mature stage, the country can live from the profits of its investments $\Rightarrow$ commercial balance deficit, yet the country is still a net creditor

It is interesting to note how the economic rise of the United States has followed the described development strategy until recently:

- For most of the nineteenth century, the USA was running a current account deficit, which was financed from abroad.
- In 1870, the country achieves a surplus on its commercial balance.
- In 1900, it achieves a current account surplus, too.
- During the first half of the twentieth century, the USA is the greatest creditor country in the world.
- In the early 1970s, the country finances the deficit on its commercial balance with the interest payments from its investments abroad.
- In the late 1970s, the current account moves into deficit, yet the country still remains a net creditor.
- In the mid-1980s, the country returns to being a net borrower.

On the other hand, many countries have not followed this pattern.

- Australia and Canada - net external borrowers during all their history

Conclusion:

- What is important is not whether a country is running a current account deficit or not,
- but that it preserves its capacity to pay its debts.
- The countries have to use the money they borrow abroad on profitable investments at home.

1.4.5 Public and private savings

$$
\begin{aligned}
CA &= S - I \\
&= S^p + S^g - I \\
&= S^p - \underbrace{(G - T)}_{\text{budget deficit}} - I.
\end{aligned}
\tag{15}
$$

Public Deficit (if everything else is held constant):

$$(G - T) \uparrow \qquad \Rightarrow \qquad CA \downarrow . \tag{16}$$

Example: USA during the 1980s (but not later).

The problem: the public spending includes mostly transfers and benefits (which are not investment expenditures). ⇒ So it can be difficult to repay the external debt.

Private saving (if everything else is held constant):

$$S^p \downarrow \quad \text{o} \quad I \uparrow \qquad \Rightarrow \qquad CA \downarrow . \tag{17}$$

Lawson doctrine:

- A current account deficit which can be attributed to the private sector is not something to worry about since it only reflects individuals' rational savings and investment decisions. (Nigel Lawson was from 1983 to 1989 Chancellor of the Exchequer in the United Kingdom.)

However, consider the case of Mexico:

- In 1994, its budget deficit did not reach 1% of GDP.
- On the other hand, its current account deficit stood at 8% of GDP.
- In 1994–1995, the country went through a severe currency crisis.

Reasons why dissaving can be worrying:

- Excessive risk-taking when private borrowers feel too secure. For example, private banks might think that the government will assist them in times of crises (bailout).
- Volatility of international capital flows (for example: portfolio investments, debt with short maturity).

2 Gains from globalization

2.1 Consumption smoothing

Let $C_1 \leq C_2$, $u'(C) > 0$ and $u''(C) < 0$. Then:

$$\frac{1}{2}u(C_1) + \frac{1}{2}u(C_2) \leq u\left(\frac{C_1 + C_2}{2}\right). \tag{18}$$

In other words, the average of the utilities of two consumption quantities is less than, or equal to, the utility of the average of the consumption quantities. The inequality is a special case of what is known as Jensen's inequality. When $C_1 < C_2$, the inequality is strict.

2.2 Derivation of the long-run budget constraint

2.2.1 The one-period budget constraint

Let W_t be the international investment position (or the net foreign assets, the net external wealth, minus the net foreign debt) of a country.

The budget constraint in period 1 is:

$$W_1 = W_0 - FA_1. \tag{19}$$

Let $KA_t = 0$ in all periods and let $CA'_t = CA - rW_{t-1}$, so that $CA'_t = TB_t + NLIA_t + NUT_t$, where $NLIA_t$ is net labour income from abroad, and $-FA_t = CA'_t + rW_{t-1}$. Then the budget constraint of period 1 becomes:

$$W_1 = (1 + r)W_0 + CA'_1. \tag{20}$$

In period 2, 3 etc., the budget constraint is:

$$W_2 = (1 + r)W_1 + CA'_2, \tag{21}$$
$$W_3 = (1 + r)W_2 + CA'_3, \tag{22}$$
$$\ldots$$

Note that we ignore changes of external wealth due to other changes in volume, price changes and exchange rate changes.

2.2.2 The intertemporal budget constraint

Now combine equations 20, 21 and 22 and all the subsequent budget constraints to obtain the intertemporal budget constraint:

$$W_t = (1+r)^t W_0 + (1+r)^{t-1} CA'_1 + (1+r)^{t-2} CA'_2 + \ldots + CA'_t. \tag{23}$$

Dividing this equation by $(1+r)^{t-1}$ yields the present-value intertemporal budget constraint:

$$\frac{1}{(1+r)^{t-1}} W_t = (1+r)W_0 + CA'_1 + \frac{1}{1+r} CA'_2 + \ldots + \frac{1}{(1+r)^{t-1}} CA'_t. \tag{24}$$

2.2.3 The long-run budget constraint

It is reasonable to assume that $[1/(1+r)^{t-1}]W_t \to 0$ when $t \to \infty$ (the so-called no-Ponzi-scheme or no-Madoff-scheme condition). Then when $t \to \infty$, the long-run budget constraint is obtained:

$$(1+r)W_0 + \mathrm{PV}_1(CA') = 0, \tag{25}$$

where

$$\mathrm{PV}_t(X) = X_t + \frac{1}{1+r}X_{t+1} + \frac{1}{(1+r)^2}X_{t+2} + \dots. \tag{26}$$

Let Y_t be gross national disposable income net of net investment income from abroad; that is, $Y_t = GDP_t + NLIA_t + NUT_t$. Since $CA'_t = Y_t - GNE_t$, the long-run budget constraint can be written as follows:

$$\mathrm{PV}_1(GNE) = (1+r)W_0 + \mathrm{PV}_1(Y) \tag{27}$$

2.2.4 Present discounted values

Useful relationships:

$$\mathrm{PV}_1(X+Y) = \mathrm{PV}_1(X) + \mathrm{PV}_1(Y), \tag{28}$$

$$\mathrm{PV}_1[\mathrm{PV}_2(X)] = \frac{1}{1+r}\,\mathrm{PV}_2(X). \tag{29}$$

For the sum of a geometric series, the following formula holds:

$$w = \sum_{i=0}^{m} a^i = 1 + a\left(\sum_{i=0}^{m} a^i\right) - a^{m+1} = 1 + aw - a^{m+1} \tag{30}$$

$$\Leftrightarrow \qquad (1-a)w = 1 - a^{m+1} \tag{31}$$

$$\Leftrightarrow \qquad w = \sum_{i=0}^{m} a^i = \frac{1-a^{m+1}}{1-a}. \tag{32}$$

Therefore, if $|a| < 1$, the sum of the geometric series converges:

$$\lim_{m\to\infty} 1 + a + a^2 + a^3 + \ldots = \lim_{m\to\infty} \sum_{i=0}^{m} a^i = \frac{1}{1-a}. \tag{33}$$

This makes it simple to calculate the present discounted value of a constant, say $\bar{X}$:

$$\begin{aligned} \mathrm{PV}_1(\bar{X}) &= \bar{X} + \frac{1}{1+r}\bar{X} + \frac{1}{(1+r)^2}\bar{X} + \dots \\ &= \frac{1}{1 - \frac{1}{1+r}}\bar{X} \\ &= \frac{1+r}{r}\bar{X}. \end{aligned} \tag{34}$$

The present discounted value of a variable X_t taking the value 0 in period 1 and the constant value $\bar{X}$ from period 2 onwards is:

$$\mathrm{PV}_1[\mathrm{PV}_2(X)] = \frac{1}{1+r}\frac{1+r}{r}\bar{X} = \frac{1}{r}\bar{X}. \tag{35}$$

2.3 Gains from consumption smoothing

2.3.1 Initial wealth

Assumptions:

- $Y_t = 0$ for all t.
- $GNE_t = C_t = \bar{C}$ for all t (smooth consumption).
- $I_t = G_t = 0$ for all t.
- $W_0 = 100$.
- $r = r^* = 0.05$.

Long-run budget constraint:

$$\mathrm{PV}_1(C) = (1+r)W_0 \tag{36}$$

$$\Leftrightarrow \quad \frac{1+r}{r}\bar{C} = (1+r)W_0 \tag{37}$$

$$\Leftrightarrow \quad \bar{C} = rW_0. \tag{38}$$

Example:

Period	Y_t	C_t	I_t	G_t	GNE_t	CA'_t	rW_{t-1}	CA_t	FA_t	W_t
0										100
1	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
2	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
3	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
4	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
5	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
6	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
7	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
8	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
9	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**
10	0	**5**	0	0	**5**	**-5**	**5**	**0**	**0**	**100**

2.3.2 Constant Y

Assumptions:

- $Y_t = \bar{Y}$ for all t.
- $GNE_t = C_t = \bar{C}$ for all t (smooth consumption).
- $I_t = G_t = 0$ for all t.
- $W_0 = 0$.
- $r = r^* = 0.05$.

Long-run budget constraint:

$$\mathrm{PV}_1(C) = \mathrm{PV}_1(Y) \tag{39}$$

$$\Leftrightarrow \quad \frac{1+r}{r}\bar{C} = \frac{1+r}{r}\bar{Y} \tag{40}$$

$$\Leftrightarrow \quad \bar{C} = \bar{Y}. \tag{41}$$

Example:

Period	Y_t	C_t	I_t	G_t	GNE_t	CA'_t	rW_{t-1}	CA_t	FA_t	W_t
0										0
1	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
2	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
3	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
4	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
5	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
6	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
7	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
8	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
9	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**
10	100	**100**	0	0	**100**	**0**	**0**	**0**	**0**	**0**

2.3.3 Income shock in period 1

Assumptions:

- $Y_1 = \bar{Y} + \Delta\bar{Y}$ for $t = 1$.
- $Y_t = \bar{Y}$ for $t = 2, 3, \ldots$.
- $GNE_t = C_t = \bar{C}$ for all t (smooth consumption).
- $I_t = G_t = 0$ for all t.
- $W_0 = 0$.
- $r = r^* = 0.05$.

Long-run budget constraint:

$$\mathrm{PV}_1(C) = \mathrm{PV}_1(Y) \tag{42}$$

$$\Leftrightarrow \quad \frac{1+r}{r}\bar{C} = \frac{1+r}{r}\bar{Y} + \Delta\bar{Y} \tag{43}$$

$$\Leftrightarrow \quad \bar{C} = \bar{Y} + \frac{r}{1+r}\Delta\bar{Y}. \tag{44}$$

Example:

Period	Y_t	C_t	I_t	G_t	GNE_t	CA'_t	rW_{t-1}	CA_t	FA_t	W_t
0										0
1	121	**101**	0	0	**101**	**20**	**0**	**20**	**-20**	**20**
2	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
3	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
4	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
5	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
6	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
7	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
8	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
9	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**
10	100	**101**	0	0	**101**	**-1**	**1**	**0**	**0**	**20**

2.3.4 Permanent income shock

Assumptions:

- $Y_t = \bar{Y} + \Delta\bar{Y}$ for $t = 1, 2, \ldots$.
- $GNE_t = C_t = \bar{C}$ for all t (smooth consumption).
- $I_t = G_t = 0$ for all t.
- $W_0 = 0$.
- $r = r^* = 0.05$.

Long-run budget constraint:

$$\mathrm{PV}_1(C) = \mathrm{PV}_1(Y) \tag{45}$$

$$\Leftrightarrow \quad \frac{1+r}{r}\bar{C} = \frac{1+r}{r}(\bar{Y} + \Delta\bar{Y}) \tag{46}$$

$$\Leftrightarrow \quad \bar{C} = \bar{Y} + \Delta\bar{Y}. \tag{47}$$

Example:

Period	Y_t	C_t	I_t	G_t	GNE_t	CA'_t	rW_{t-1}	CA_t	FA_t	W_t
0										0
1	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
2	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
3	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
4	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
5	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
6	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
7	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
8	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
9	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**
10	101	**101**	0	0	**101**	**0**	**0**	**0**	**0**	**0**

2.4 Gains from investment

Assumptions:

- $Y_1 = \bar{Y}$ for $t = 1$.
- $Y_t = \bar{Y} + \Delta\bar{Y}$ for $t = 2, 3, \ldots$
- $C_t = \bar{C}$ for all t (smooth consumption).
- $I_1 = \bar{I}$.
- $I_t = 0$ for $t = 2, 3, \ldots$
- $G_t = 0$ for all t.
- $W_0 = 0$.
- $r = r^* = 0.05$.

If investment is carried out, $\bar{I} > 0$ and $\Delta\bar{Y} > 0$. Otherwise $\bar{I} = 0$ and $\Delta\bar{Y} = 0$.

Long-run budget constraint:

$$\mathrm{PV}_1(C) + \mathrm{PV}_1(I) = \mathrm{PV}_1(Y) \tag{48}$$

$$\Leftrightarrow \quad \frac{1+r}{r}\bar{C} + \bar{I} = \frac{1+r}{r}\bar{Y} + \frac{1}{1+r}\frac{1+r}{r}\Delta\bar{Y} \tag{49}$$

$$\Leftrightarrow \quad \bar{C} = \frac{r}{1+r}\left[\frac{1+r}{r}\bar{Y} + \frac{1}{r}\Delta\bar{Y} - \bar{I}\right]. \tag{50}$$

The investment is carried out if:

$$\mathrm{PV}_1(\Delta Y) > \mathrm{PV}_1(I) \tag{51}$$

$$\Leftrightarrow \quad \frac{1}{r}\Delta\bar{Y} > \bar{I}. \tag{52}$$

Example:

Period	Y_t	C_t	I_t	G_t	GNE_t	CA'_t	rW_{t-1}	CA_t	FA_t	W_t
0										0
1	100	**101**	79	0	**180**	**-80**	**0**	**-80**	**80**	**-80**
2	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
3	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
4	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
5	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
6	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
7	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
8	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
9	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**
10	105	**101**	0	0	**101**	**4**	**-4**	**0**	**0**	**-80**

2.5 External debt reduction

The variables W_0 and $\bar{Y}$ are given. The variables T and W_T have to be specified. Then $\bar{C}$ can be calculated as follows:

$$(1+r)W_0 + Y_1 + \frac{1}{1+r}Y_2 + \ldots + \frac{1}{(1+r)^{T-1}}Y_T = C_1 + \frac{1}{1+r}C_2 + \ldots + \frac{1}{(1+r)^{T-1}}C_T + \frac{1}{(1+r)^{T-1}}W_T \quad (53)$$

$$\Leftrightarrow \quad (1+r)W_0 + \bar{Y} + \frac{1}{1+r}\bar{Y} + \ldots + \frac{1}{(1+r)^{T-1}}\bar{Y} = \bar{C} + \frac{1}{1+r}\bar{C} + \ldots + \frac{1}{(1+r)^{T-1}}\bar{C} + \frac{1}{(1+r)^{T-1}}W_T \quad (54)$$

$$\Leftrightarrow \quad (1+r)W_0 + \mathrm{PV}_1(\bar{Y}) - \frac{1}{(1+r)^T}\,\mathrm{PV}_{T+1}(\bar{Y}) = \mathrm{PV}_1(\bar{C}) - \frac{1}{(1+r)^T}\,\mathrm{PV}_{T+1}(\bar{C}) + \frac{1}{(1+r)^{T-1}}W_T \quad (55)$$

$$\Leftrightarrow \quad (1+r)W_0 + \left(1 - \frac{1}{(1+r)^T}\right)\frac{1+r}{r}\bar{Y} = \left(1 - \frac{1}{(1+r)^T}\right)\frac{1+r}{r}\bar{C} + \frac{1}{(1+r)^{T-1}}W_T \tag{56}$$

$$\Leftrightarrow \quad W_0 + \left(1 - \frac{1}{(1+r)^T}\right)\frac{1}{r}\bar{Y} = \left(1 - \frac{1}{(1+r)^T}\right)\frac{1}{r}\bar{C} + \frac{1}{(1+r)^T}W_T \tag{57}$$

$$\Leftrightarrow \quad \bar{C} = \bar{Y} + r\left(\frac{W_0 - \frac{1}{(1+r)^T}W_T}{1 - \frac{1}{(1+r)^T}}\right). \tag{58}$$

3 Introduction to exchange rates

3.1 The Foreign Exchange Market

3.1.1 Nominal Exchange Rate

The exchange rate is the price of one currency in terms of another currency.

Euro exchange rate:

- In direct terms: 0.80 € for 1.00 $
- In indirect terms: 1.25 $ for 1.00 €

Exchange rate changes:

Exchange rate	in direct terms	in indirect terms
Depreciation	$\uparrow$	$\downarrow$
Appreciation	$\downarrow$	$\uparrow$

From now on, unless otherwise indicated, we will use exchange rates in indirect terms.

Price of exports and imports:

	Price of exports (for foreigners)	Price of imports (for us)
Depreciation	$\downarrow$	$\uparrow$
Appreciation	$\uparrow$	$\downarrow$

Effect on the demand for exports and imports:

	Demand for exports (to the foreign country)	Demand for imports (to our country)
Depreciation	$\uparrow$	$\downarrow$
Appreciation	$\downarrow$	$\uparrow$

3.1.2 Real Exchange Rate

The real exchange rate (in indirect terms) compares the price level in our country with the price level in the foreign country after converting the price levels into the same currency so as to make them comparable:

$$Q = \frac{SP}{P^*}\frac{[\$]}{[\$]} = \frac{P}{\frac{1}{S}P^*}\frac{[€]}{[€]}. \tag{59}$$

3.1.3 Balance of payments adjustment theories

J-curve effect

To simplify, suppose that the current account equals net exports:

$$z = X - \frac{1}{S}M, \tag{60}$$

where

$$\begin{aligned} z &= \text{current account}, \\ X &= \text{exports (in the domestic currency)} \\ M &= \text{imports (in the foreign currency)} \end{aligned} \tag{61}$$

In the short run:

$$S \downarrow \quad \Rightarrow \quad \frac{1}{S}M \uparrow \quad \Rightarrow \quad z \downarrow \qquad \text{(effect on the value).} \tag{62}$$

In the long run:

$$S \downarrow \quad \Rightarrow \quad X \uparrow, M \downarrow \quad \Rightarrow \quad z \uparrow \qquad \text{(effect on the quantity).} \tag{63}$$

If the **Marshall-Lerner condition** holds, the **long run effect is stronger than the short run effect.**

Taking the derivative with respect to S, we obtain:

$$\frac{\partial z}{\partial S} = \frac{\partial X}{\partial S} - \frac{1}{S}\frac{\partial M}{\partial S} - \left(-\frac{1}{S^2}\right) M \quad \Leftrightarrow \quad \frac{\partial z}{\partial S}\frac{S}{X} = \frac{\partial X}{\partial S}\frac{S}{X} - \frac{1}{S}\frac{\partial M}{\partial S}\frac{S}{X} + \frac{1}{S^2} M \frac{S}{X}. \tag{64}$$

In equilibrium, $X = \frac{M}{S}$:

$$\frac{\partial z}{\partial S}\frac{S}{X} = \frac{\partial X}{\partial S}\frac{S}{X} - \frac{\partial M}{\partial S}\frac{S}{M} + 1 = \eta_{X,S} - \eta_{M,S} + 1 < 0. \tag{65}$$

Then we obtain the Marshall-Lerner condition:

$$|\eta_{X,S}| + \eta_{M,S} > 1. \tag{66}$$

The **Marshall-Lerner condition** demonstrates that for a currency devaluation to have a positive impact on the trade balance, **the sum of the price elasticities of imports and exports must be, in absolute value, greater than 1.**

Empirically, the following has been demonstrated:

- **In the short run,** goods tend to be inelastic, since it takes time to change consumption patterns. Therefore **the Marshall-Lerner condition does not hold** and initially a devaluation will worsen the trade balance.
- **In the long run,** however, consumers do adjust to new prices and **the trade balance does improve.**

This gives rise to the **J curve,** which is named after the evolution of the trade balance after an exchange rate devaluation in a diagram with the time on the horizontal axis and the trade balance on the vertical axis.

3.1.4 The foreign exchange market

Agents:

- Commercial banks
  - Interbank operations (wholesale market)
  - Client operations (retail market, less favourable terms)
  - Profit = retail market - wholesale market
- Multinational companies
- non-bank financial institutions
  - Institutional investors, for example pension funds
  - Investment funds
  - Insurance companies
- Central banks

Market characteristics:

- Daily turnover in the worldwide foreign exchange market:
  - 1989: 0.6 trillion USD per day
  - 2004: 1.9 trillion USD per day
  - 2007: 4.0 trillion USD per day (de.wikipedia.org)
- Main places:
  - London, New York, Tokyo, Frankfurt
- World currencies:
  - US dollar (USA)
  - Euro (EU)

3.1.5 Spot and forward exchange rates

- Spot exchange rate
- Forward exchange rate
- Swap exchange rate

Currency swap:

- to exchange a given amount of one currency for another and,
- after a specified period of time, to re-exchange the principal amount at the maturity of the deal (with an adjustment made to compensate for changes in the principal value).

3.1.6 Derivative products

- Futures contract
- Foreign exchange options (put option, call option)

3.2 The euro

3.2.1 European Monetary Union

See www.wikipedia.org.

The **European Monetary Union** was the outcome of a process with three important stages:

- **1990: Complete liberalization of capital markets,** participation in the **European Monetary System (EMS)** and presentation of the economic convergence programmes.
- **1994:** The **European Monetary Institute** starts to operate as an intermediate step before the implementation of the **European System of Central Banks (ESCB).**
- **1999: Introduction of the euro.** On 1 January 1999 the new European currency is put into circulation, but only virtually, no notes and coins are used yet.
- **2002:** On 1 January **the euro starts to circulate** in twelve of the member states.

Convergence criteria.

See www.wikipedia.org.

The **convergence criteria,** or **Maastricht criteria,** are the requirements that the member states of the European Union must satisfy **in order to be admitted to the eurozone** and thereby to participate in the Eurosystem. In total there are **four criteria:**

- **Public finance:** The following two criteria must be met:
  - On one hand the **budget deficit** of the public administration **must not represent more than 3% of GDP** at the end of the previous year. A country with a deficit greater than 3% may exceptionally be admitted as long as its budget deficit stays close to the limit and there is the perspective of a reduction in the near future.
  - On the other hand, **public debt must not exceed 60% of GDP.** If public debt represents more than 60% of national income, the country can still be admitted in the eurozone as long as the ratio converges to, and stays close to, the limit. In practice, this criterion is usually omitted when a country is admitted to the eurozone, because when the euro was created there were many states that did not comply with the criterion.

- **Inflation rate:** The inflation rate **must not be 1.5 percentage points higher than the average of the three member states with the lowest inflation** (excluding those with deflation) during the year before review.
- **Exchange rate:** The state must participate in the **European Monetary System (EMS)** exchange rate mechanism without any break **during two years** preceding the review and with no serious conflicts. Furthermore, it should not have devaluated its currency during the same period. After the transition to the third stage of the EMS, the European Monetary System was replaced by the new exchange rate mechanism (ERM II).
- **Long term nominal interest rate:** The nominal long-term interest rate **must not be more than 2 percentage points higher than in the three member states with the lowest inflation** during the year before the review.

3.2.2 The evolution of the euro against the dollar

- The **euro** is the **the currency of the eurozone, formed in 2015 by 19 out of the 28 member states of the EU** that share the common currency.
- The euro was **introduced on 1 January 1999.** However, due to the time required for manufacturing the new banknotes and coins, the old national currencies, despite of losing its official quotation in the foreign exchange market, could still for payments.
- **The euro notes and coins** started to circulate on **1 January 2002,** and at that time one euro was exchanged for **0.9038 US dollars (USD).**
- Since then, the euro's value against the US dollar evolved as follows:

Date	Exchange rate ($ per €)	Comment
01/01/1999	1.1789	Introduction of the euro
27/01/2000	1.0000	Parity between euro and dollar
26/10/2000	0.8252	Minimum value
01/01/2002	0.9038	Introduction of banknotes and coins
Julio de 2002	1.0000	Parity between euro and dollar
15/07/2008	1.5990	Maximum value

4 Basic financial concepts

4.1 Return and risk

4.1.1 Logarithms and percentages

The natural logarithm is the inverse of the exponential function:

$$\ln(e^x) = x. \tag{67}$$

Therefore $e^0 = 1$ implies that $\ln(1) = 0$ for instance.

Since the natural logarithm is the inverse of the exponential function, the exponential function is also the inverse of the logarithmic function:

$$x = e^{\ln x} \qquad (x > 0). \tag{68}$$

Properties of the logarithm:

$$\begin{aligned} \ln(a \times b) &= \ln\left(e^{\ln(a)} \times e^{\ln(b)}\right) \\ &= \ln\left(e^{\ln(a)+\ln(b)}\right) \\ &= \ln(a) + \ln(b), \end{aligned} \tag{69}$$

$$\begin{aligned}\ln(x^a) &= \ln\left(\left(e^{\ln(x)}\right)^a\right)\\ &= \ln\left(e^{a\ln(x)}\right)\\ &= a\ln(x),\end{aligned} \tag{70}$$

$$x^a = \left(e^{\ln x}\right)^a = e^{a\ln x}. \tag{71}$$

What about the logarithm to a base different from e? Consider the logarithm to the base a, $\log_a(x)$. We can find a formula for $\log_a(x)$ as follows:

$$a^{\log_a(x)} = x \tag{72}$$

$$\Leftrightarrow \quad \ln\left(a^{\log_a(x)}\right) = \ln(x) \tag{73}$$

$$\Leftrightarrow \quad \log_a(x)\ln(a) = \ln(x) \tag{74}$$

$$\Leftrightarrow \quad \log_a(x) = \frac{\ln(x)}{\ln(a)}. \tag{75}$$

Hence, for example:

$$
\begin{aligned}
\log_2(x) &= \frac{\ln(x)}{\ln(2)} \approx \frac{\ln(x)}{0.69} \approx 1.44 \ln(x), \\
\log_{10}(x) &= \frac{\ln(x)}{\ln(10)} \approx \frac{\ln(x)}{2.30} \approx 0.43 \ln(x).
\end{aligned}
\tag{76}
$$

Therefore, if we plot a logarithmic time series, the plot will look the same regardless of the base of the logarithm. The only thing that will change is the scale of the graph.

The first-order Taylor approximation of the natural logarithm is given by:

$$\ln(x) \approx \ln(x_0) + \left.\frac{d\ln(x)}{dx}\right|_{x=x_0} \times (x - x_0). \tag{77}$$

Now let $x_0 = 1$ and $x - x_0 = a$. If a is small, it holds that:

$$\begin{aligned}\ln(1+a) &= \ln(x)\\ &\approx \ln(x_0) + \left.\frac{d\ln(x)}{dx}\right|_{x=x_0} \times (x - x_0)\\ &= \ln(1) + \left.\frac{1}{x}\right|_{x=1} \times a\\ &= a.\end{aligned} \tag{78}$$

We therefore have the following useful approxmiations:

$$\ln(1+a) \approx a \tag{79}$$

$$\ln(1-a) \approx -a \tag{80}$$

$$\ln\left(\frac{1+a}{1+b}\right) = \ln(1+a) - \ln(1+b) \approx a - b \tag{81}$$

It immediately follows that:

$$e^a \approx 1 + a, \tag{82}$$

$$e^{-a} \approx 1 - a, \tag{83}$$

$$\frac{1+a}{1+b} = e^{a-b} \approx 1 + a - b. \tag{84}$$

Log-differencing:

If X is a variable and x its natural logarithm, that is, if $\ln(X) = x$, then we find that the absolute change in x is approximately equal to the percentage change of X. This gives rise to the method of ”log-differencing”:

$$\Delta x_t = x_t - x_{t-1} = \ln\left(\frac{X_t}{X_{t-1}}\right) = \ln\left(1 + \frac{X_t - X_{t-1}}{X_{t-1}}\right) \approx \frac{X_t - X_{t-1}}{X_{t-1}}. \tag{85}$$

4.1.2 Demand for financial assets

Important considerations when making an investment in financial assets (for example in currencies):

- Return
- Risk
- Liquidity

Real return versus nominal return:

- Gross nominal return: $1 + R$
- Net nominal return: R
- Gross real return: $\frac{1+R}{1+\pi} \approx 1 + R - \pi$
- Net real return: $\frac{1+R}{1+\pi} - 1 \approx R - \pi$

Return	Type of asset	Risk
More	Options and other derivatives	More
	Shares in developing countries	
	Commodities	
	Real estate (financed by mortgages)	
	Shares in developed countries	
	Speculative-grade bonds, issued by countries or companies (”junk bonds”)	
	Corporate bonds	
	Real estate (financed with own capital)	
	Long-term German government bonds	
	Life insurances (classified according to return)	
Less	Fixed-term deposits and German government bonds with shorter maturity	Less

4.1.3 Return and the time it takes for an asset's value to double

The **time it takes for an asset's value to double** can be estimated as follows (where g is the net return):

$$(1+g)^t = 2 \tag{86}$$

$$\Leftrightarrow \quad t\ln(1+g) = \ln 2 \tag{87}$$

$$\Leftrightarrow \quad t = \frac{\ln 2}{\ln(1+g)} = \frac{0{,}693}{\ln(1+g)}. \tag{88}$$

We know that $\ln(1+g) \approx g$. Then:

$$t \approx \frac{69{,}3}{100 \times g}. \tag{89}$$

But we know that always $\ln(1+g) < g$. Therefore we could replace the previous equation's numerator with 72, a number that is a little bit higher than 69.3 and that has the advantage that it can be easily divided by 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72:

$$t \approx \frac{72}{100 \times g}. \tag{90}$$

This is the **law of the number 72.**

Another thing we are sometimes interested in is to know **how much an asset must grow in order to double in t periods.** The exact formula is:

$$(1+g)^t = 2 \tag{91}$$
$$\Leftrightarrow \quad 1+g = \sqrt[t]{2} \tag{92}$$
$$\Leftrightarrow \quad g = \sqrt[t]{2} - 1. \tag{93}$$

But of course we can also use the above approximation:

$$g \approx \frac{72}{100 \times t}. \tag{94}$$

4.2 Asset returns and asset prices

- An asset's return can be estimated ”ex ante” and ”ex post”.
- Ex ante, only variables up to period t are known; however, the values of variables in the future have to be forecast.

The general interest rate.

The general interest rate of period $t+1$ (that is, between the end of t and the end of $t+1$) is:

$$R_{t+1}. \tag{95}$$

Bonds.

The ex-ante net return of a market bond between the end of t and the end of $t+1$ is (B = bond):

$$R_{t+1}^{B}=\frac{P_{t+1}^{B}-P_{t}^{B}+C_{t+1}P_{0}^{B}}{P_{t}^{B}}, \tag{96}$$

where C_{t+1} is the coupon rate of period $t+1$ fixed at date 0.

For example, if the current price of a bond that yields a coupon of 5% is 1.12 and the predicted price in the next period is 1.10, then the return is:

$$R_{t+1}^{B}=\frac{1.10-1.12+5\%\times 1.00}{1.12}=0.027. \tag{97}$$

In this example, the ex-ante net return would only be 2.7%, even though the coupon rate is 5%.

Price of a bond based on its future price, P^B_{t+1}, and the interest rate, R_{t+1}.

Note that the current price of the bond, P^B_t, can be estimated on the basis of its future price, P^B_{t+1}, and the general interest rate, R_{t+1}:

$$R^B_{t+1} = R_{t+1} \tag{98}$$

$$\Leftrightarrow \frac{P^B_{t+1} - P^B_t + C_{t+1}P^B_0}{P^B_t} = R_{t+1} \tag{99}$$

$$\Leftrightarrow P^B_t = \frac{P^B_{t+1} + C_{t+1}P^B_0}{1 + R_{t+1}}. \tag{100}$$

Supposing that $P^B_0 = 1.00$, we have:

$$P^B_t = \frac{P^B_{t+1} + C_{t+1}}{1 + R_{t+1}}. \tag{101}$$

As at the end of period t the values of C_{t+1} and R_{t+1} are known, we conclude that the current price of a bond, P^B_t, depends only on a single unknown variable, which is P^B_{t+1}. For example, if P^B_{t+1} increases by one euro, P^B_t rises $1/(1 + R_{t+1})$ (≈ 1) euros.

On the other hand, if we suppose that expectations about the future price, P_{t+1}^B, do not change, we observe that the interest rate, R_{t+1}, has a negative effect on the current price of the bond, P_t^B:

$$\frac{\partial P_t^B}{\partial R_{t+1}} = -\frac{P_{t+1}^B + C_{t+1}}{(1+R_{t+1})^2} < 0. \tag{102}$$

For example, if $P_0^B = 1.00$, $P_{t+1}^B = 1.10$ and $C_{t+1} = R_{t+1} = 0.05$, then an increase in the interest rate, R_t, of 0.01 (maybe provoked by the central bank) leads to a reduction of the current price of the bond, P_t^B, of 0.0104 units.

Bonds (last period).

The ex-ante net return of a market bond between $T-1$ and T is:

$$R_T^B = \frac{P_T^B - P_{T-1}^B + C_T P_0^B}{P_{T-1}^B}. \tag{103}$$

Hence we see that the price of the bond in $T-1$ also depends on its price in T:

$$\begin{aligned} P_{T-1}^B &= \frac{P_T^B + C_T P_T^B}{1+R_T^B} \\ &= \frac{1+C_T}{1+R_T^B} P_0^B \\ &\approx 1 + C_T^B - R_T, \end{aligned} \tag{104}$$

where it is assumed that $P_0^B = 1$ and where we have ignored the issue of risk, so that in the absence of arbitrage $R_T^B = R_T$.

Using P^B_{T-1}, we can compute P^B_{T-2}:

$$R^B_{T-1} = \frac{P^B_{T-1} - P^B_{T-2} + C_{T-1}P^B_0}{P^B_{T-2}} \tag{105}$$

$$\Leftrightarrow \quad P^B_{T-2} = \frac{1 + C_T - R_T + C_{T-1}}{1 + R_{T-1}} \approx 1 + C_T - R_T + C_{T-1} - R_{T-1}. \tag{106}$$

Continuing in this fashion, we can recursively compute the bond prices for $T-3, T-4, \ldots, T-k$:

$$P^B_{T-k} = 1 + \sum_{i=0}^{k-1}(C_{T-i} - R_{T-i}). \tag{107}$$

Shares.

The ex-ante net return of a share between t and $t+1$ is (S = share):

$$R^S_{t+1} = \frac{P^S_{t+1} - P^S_t + D_{t+1}}{P^S_t}. \tag{108}$$

The variable D_{t+1} is the dividend which the investor receives at the end of the period $t+1$.

For example, if the current price of a share is 112.00 and we predict a price in the next period of 110.00 and a dividend of 3.00, then the return is:

$$\frac{110.00 - 112.00 + 3.00}{112.00} = 0.009. \tag{109}$$

In this example, the net ex-ante return would be 0.9%.

Ignoring the issue of risk and using the no-arbitrage condition $R^S_{t+1} = R_{t+1}$, we can derive a formula for the valuation of shares based on the present discounted value of all future dividends:

$$
\begin{aligned}
P^S_t &= \frac{P^S_{t+1} + D_{t+1}}{1 + R_{t+1}} \\
&= \frac{D_{t+1}}{1 + R_{t+1}} + \frac{P^S_{t+2} + D_{t+2}}{(1 + R_{t+1})(1 + R_{t+2})} \\
&= \dots \\
&= \sum_{j=1}^{k} \frac{D_{t+j}}{\prod_{i=1}^{j}(1 + R_{t+i})}.
\end{aligned} \tag{110}
$$

Real estate (houses and offices).

The ex-ante net return of a real estate (a house or an office) between t and $t+1$ is (H = house):

$$R_{t+1}^{H} = \frac{P_{t+1}^{H} - P_{t}^{H} + H_{t+1}}{P_{t}^{H}}. \tag{111}$$

The variable H_{t+1} is the rent that the investor receives at the end of the period $t+1$.

For example, if the current price of a real estate is 300.000, the yearly rent is 18.000 and we predict a price of the real estate of 290.000 in the next period, then the return is:

$$\frac{290.000 - 300.000 + 18.000}{300.000} = 0.027. \tag{112}$$

In this example, the ex-ante net return would be 2.7%.

Commodities (for example gold).

The ex-ante net return of an ounce of gold between t and $t+1$ is (M = metal):

$$R_{t+1}^{M} = \frac{P_{t+1}^{M} - P_{t}^{M}}{P_{t}^{M}}. \tag{113}$$

For example, if the current price of an ounce of gold is 886.00 and the predicted price is 905.00 in the next period, then the return is:

$$\frac{905.00 - 886.00}{886.00} = 0.021. \tag{114}$$

In this example, the ex-ante net return would be 2.1%.

4.3 Leverage

For example, **a house is purchased** with the following return:

$$\frac{P_{t+1}^H - P_t^H + R_t^H}{P_t^H} - \omega_t^H. \tag{115}$$

Let P_t^H be equal to 500,000 €.

There are two possibilities:

- **Option A:** The house is purchased with one's **own money.**
- **Option B:** The house is purchased after **borrowing 80% of the money** from a bank (with a mortgage interest rate of 4%).

The return and the risk of each one of the options can be determined with the following chart (ignoring R_t^H and ω_t^H for simplicity):

Option	$\frac{P^H_{t+1}-P^H_t}{P^H_t}$	$P^H_{t+1} - P^H_t$	Mortgage interest	Net profit	Return	
A	+20%	100,000 €	0 €	100,000 €	+20%	
	+8%	40,000 €	0 €	40,000 €	+8%	
	+2%	10,000 €	0 €	10,000 €	+2%	
	-8%	-40,000 €	0 €	-40,000 €	-8%	
	-20%	-100,000 €	0 €	-100,000 €	-20%	
B	+20%	100,000 €	-16,000 €	84,000 €	+84%	(!)
	+8%	40,000 €	-16,000 €	24,000 €	+24%	
	+2%	10,000 €	-16,000 €	-6,000 €	-6%	(!)
	-8%	-40,000 €	-16,000 €	-56,000 €	-56%	
	-20%	-100,000 €	-16,000 €	-116,000 €	-116%	(!)

4.4 Return parities

In theory, the ex-ante returns (including the risk premia) must be the same for all assets:

$$\begin{aligned} R_t^B - \omega_t^B &= R_t^S - \omega_t^S \\ &= R_t^H - \omega_t^H \\ &= R_t^C - \omega_t^C \\ &= R_t - \omega_t, \end{aligned} \tag{116}$$

where $\omega_t^{\cdot}$ represents the risk premium of the respective asset class.

Typically, central banks aim to influence the short-term interest rate, R_t, for example through open-market operations. Given what we have just seen, it is evident that a change in R_t has important and immediate effects on the returns of the alternative assets (bonds, stocks, real estate, commodities) and their prices.

4.5 Interest rate parity

Comparison of investments in domestic and foreign assets (investment of 1 €):

$$1 + R = S(1 + R^*)\frac{1}{S^e} \tag{117}$$

$$\Leftrightarrow \quad \ln(1 + R) = \ln(1 + R^*) + \ln(S) - \ln(S^e) \tag{118}$$

$$\Rightarrow \quad R \approx R^* + s - s^e \tag{119}$$

$$\Rightarrow \quad R \approx R^* + \frac{S - S^e}{S^e}. \tag{120}$$

The foreign exchange market is in equilibrium when the deposits of all the currencies offer the same expected rate of return.

Uncovered interest parity:

$$R = R^* + \underbrace{\frac{S - S^e}{S^e}}_{\text{depreciation rate}} . \tag{121}$$

Covered interest parity (derived analogously):

$$R = R^* + \underbrace{\frac{S - F}{F}}_{\text{depreciation rate}} . \tag{122}$$

Here F is the forward exchange rate.

4.6 International investment

4.6.1 Expectation, variance, covariance and correlation

Let X, Y and Z be random variables and let a and c be constants.

Expectation:

$$\mathrm{E}(X) = \sum_i x_i p(x_i), \qquad \text{when } X \text{ takes discrete values,} \tag{123}$$

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} x f(x) dx, \qquad \text{when } X \text{ takes continuous values.} \tag{124}$$

Thus we have, for example:

$$\mathrm{E}(aX) = a\,\mathrm{E}(X), \tag{125}$$

$$\mathrm{E}(c) = c, \tag{126}$$

$$\mathrm{E}(X+Y) = \mathrm{E}(X) + \mathrm{E}(Y). \tag{127}$$

Covariance:

$$\begin{aligned} \mathrm{Cov}(X,Y) &= \mathrm{E}[(X - \mathrm{E}(X))(Y - \mathrm{E}(Y))] \\ &= \mathrm{E}(XY) - \mathrm{E}(X)\,\mathrm{E}(Y). \end{aligned} \tag{128}$$

Thus we have, for example:

$$\mathrm{Cov}(aX, Y) = a\,\mathrm{Cov}(X, Y), \tag{129}$$

$$\mathrm{Cov}(X, c) = 0, \tag{130}$$

$$\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z). \tag{131}$$

Variance:

$$\begin{aligned}\operatorname{Var}(X) &= \operatorname{Cov}(X, X), \\ &= \mathrm{E}[(X - \mathrm{E}(X))^2] \\ &= \mathrm{E}(X^2) - \mathrm{E}(X)^2.\end{aligned} \tag{132}$$

Thus we have, for example:

$$\operatorname{Var}(c) = 0, \tag{133}$$

$$\operatorname{Var}(aX) = a^2 \operatorname{Var}(X), \tag{134}$$

$$\operatorname{Var}(X + c) = \operatorname{Var}(X), \tag{135}$$

$$\operatorname{Var}(X + Y) = \operatorname{Var}(X) + \operatorname{Var}(Y) + 2 \operatorname{Cov}(X, Y), \tag{136}$$

$$\operatorname{Var}(X - Y) = \operatorname{Var}(X) + \operatorname{Var}(Y) - 2 \operatorname{Cov}(X, Y). \tag{137}$$

Correlation:

$$\operatorname{Corr}(X, Y) = \frac{\operatorname{Cov}(X, Y)}{\sqrt{\operatorname{Var}(X) \operatorname{Var}(Y)}}. \tag{138}$$

4.6.2 Optimization of an international portfolio

An investor has wealth W and can choose between three assets:

- safe bond with expected return $\mathrm{E}(\bar{R}) = \bar{\mu}$ and variance $\mathrm{Var}(\bar{R}) = 0$,
- domestic bond with expected return $\mathrm{E}(R) = \mu$ and variance $\mathrm{Var}(R) = \sigma^2$,
- foreign bond with expected return $\mathrm{E}(R^*) = \mu^*$ and variance $\mathrm{Var}(R^*) = (\sigma^*)^2$.

The investor invests a share w of her or his wealth in domestic bonds, a share w^* in foreign bonds and a share $(1 - w - w^*)$ in safe bonds.

The correlation between the returns of the domestic and foreign bonds is:

$$\rho = \frac{\mathrm{Cov}(R, R^*)}{\sigma\sigma^*}. \tag{139}$$

Expected value of the portfolio:

$$\mathrm{E}[wWR + w^*WR^* + (1-w-w^*)W\bar{R}] = wW\mu + w^*W\mu^* + (1-w-w^*)W\bar{\mu}. \tag{140}$$

Variance of the portfolio:

$$\mathrm{Var}[wWR + w^*WR^* + (1-w-w^*)W\bar{R}] = w^2W^2\sigma^2 + (w^*)^2W^2(\sigma^*)^2 + 2ww^*W^2\sigma\sigma^*\rho. \tag{141}$$

To optimize the portfolio, it is necessary to strike a balance between return and risk:

$$\begin{aligned} \max_{w,w^*} \quad & wW\mu + w^*W\mu^* + (1-w-w^*)W\bar{\mu} \\ & - \frac{1}{2}\lambda\left[w^2W^2\sigma^2 + (w^*)^2W^2(\sigma^*)^2 + 2ww^*W^2\sigma\sigma^*\rho\right]. \end{aligned} \tag{142}$$

The coefficient λ measures the cost of the risk relative to the return.

Now let us equalize the first derivatives with respect to w and w^* with zero.

$$\mu - \bar{\mu} - \lambda[wW\sigma^2 + w^*W\sigma\sigma^*\rho] = 0, \tag{143}$$
$$\mu^* - \bar{\mu} - \lambda[w^*W(\sigma^*)^2 + wW\sigma\sigma^*\rho] = 0. \tag{144}$$

We thus obtain:

$$wW = \frac{(\mu - \bar{\mu}) - (\mu^* - \bar{\mu})\frac{\sigma}{\sigma^*}\rho}{\lambda\sigma^2(1-\rho^2)}, \tag{145}$$
$$w^*W = \frac{(\mu^* - \bar{\mu}) - (\mu - \bar{\mu})\frac{\sigma^*}{\sigma}\rho}{\lambda(\sigma^*)^2(1-\rho^2)}. \tag{146}$$

Note that w and w^* can take negative values (short-selling).

In the following table, it is supposed that $W = 1$, $\bar{\mu} = 2.5$, $\mu = 3$, $\mu^* = 5$, $\sigma^2 = 1$ ($\sigma = 1$) and $(\sigma^*)^2 = 4$ ($\sigma^* = 2$).

Note that $\mathrm{SD}(R^{\mathrm{opt}})$ refers to the standard deviation (or square root of the variance) of the return on the optimal portfolio.

λ	ρ	$\bar{w}$	w	w^*	$\mathrm{E}(R^{\mathrm{opt}})$	$\mathbf{SD}(R^{\mathrm{opt}})$
0.40	−0.80	−15.15	10.42	5.73	22.03	6.99
0.40	−0.40	−4.13	2.98	2.16	9.38	4.15
0.40	0.00	−1.81	1.25	1.56	7.03	3.37
0.40	0.40	−0.56	0.00	1.56	6.41	3.13
0.40	0.80	1.52	−3.47	2.95	8.14	3.76
1.00	−0.80	−5.46	4.17	2.29	10.31	2.80
1.00	−0.40	−1.05	1.19	0.86	5.25	1.66
1.00	0.00	−0.13	0.50	0.63	4.31	1.35
1.00	0.40	0.38	0.00	0.63	4.06	1.25
1.00	0.80	1.21	−1.39	1.18	4.76	1.50
2.50	−0.80	−1.58	1.67	0.92	5.63	1.12
2.50	−0.40	0.18	0.48	0.35	3.60	0.66
2.50	0.00	0.55	0.20	0.25	3.23	0.54
2.50	0.40	0.75	0.00	0.25	3.13	0.50
2.50	0.80	1.08	−0.56	0.47	3.40	0.60

5 Monetary models of exchange rate determination

5.1 Money

5.1.1 Definition of money

Money includes currency and (liquid) bank deposits.

Characteristics:

- Medium of exchange
- Unit of account
- Store of value

Comparison with other financial assets:

	Money	Other assets
Liquidity	+	-
Return	-	+

5.1.2 Money multiplier

Balance sheet of the central bank:

Assets	Liabilities
Bonds (D)	Bank reserves (BR)
Offcial reserves(OR)	Currency (coins and bank notes) in circulation (CU)

The assets represent the base money supply, H^s, and the liabilities the base money demand, H^d:

$$H^s = D + OR, \tag{147}$$

$$H^d = CU^d + BR^d. \tag{148}$$

The monetary base, H, is also called high-powered money.

Balance sheet of the banking sector:

Assets	Liabilities
Bank reserves (BR)	Bank deposits (BD)
Bonds and loans (B)	

Money demand:

$$M^d = PL(Y,R) = CU^d + BD^d, \tag{149}$$
$$CU^d = cM^d, \tag{150}$$
$$BD^d = (1-c)M^d. \tag{151}$$

Under fractional reserve banking, in which θ is the reserve ratio (or reserve requirement):

$$BR = \theta(1-c)M^d = \theta BD. \tag{152}$$

Therefore:

$$\begin{aligned} H^d &= CU^d + BR^d \\ &= [c+\theta(1-c)]M^d \\ &= [c+\theta(1-c)]M^s \\ &= H^s \end{aligned} \tag{153}$$

$$\Leftrightarrow \quad M^s = \frac{1}{c+\theta(1-c)}H^s. \tag{154}$$

The quotient $1/[c+\theta(1-c)]$ is the money multiplier.

Example 1: The central bank buys bonds of a value of $100.00€$. To simplify, suppose that $c = 0$ (in reality, c is around 0.40). If $\theta = 0.10$, then the money multiplier is $1/\theta = 10$:

$$100 + (1 - \theta) \times 100 + (1 - \theta)^2 \times 100 + (1 - \theta)^3 \times 100 + \ldots = \frac{1}{1 - (1 - \theta)} = \frac{1}{\theta}. \tag{155}$$

BD	BR	B
$+100.00€$	$+10.00€$	$+90.00€$
$+90.00€$	$+9.00€$	$+81.00€$
$+81.00€$	$+8.10€$	$+72.90€$
$+72.9€$	$+7.29€$	$+65.61€$
$\vdots$	$\vdots$	$\vdots$
$+1000.00€$	$+100.00€$	$+900.00€$

Example 2: Now suppose that $c > 0$. We then have:

BD	BR	B
		$+1$
$+(1-c)$	$+\theta(1-c)$	$+(1-c)-\theta(1-c)$
$+(1-c)[(1-c)-\theta(1-c)]$	$+\theta(1-c)[(1-c)-\theta(1-c)]$	$+[(1-c)-\theta(1-c)]^2$
$+(1-c)[(1-c)-\theta(1-c)]^2$	$+\theta(1-c)[(1-c)-\theta(1-c)]^2$	$+[(1-c)-\theta(1-c)]^3$
$+(1-c)[(1-c)-\theta(1-c)]^3$	$+\theta(1-c)[(1-c)-\theta(1-c)]^3$	$+[(1-c)-\theta(1-c)]^4$

Money multiplier:

$$1+[(1-c)-\theta(1-c)]+[(1-c)-\theta(1-c)]^2+[(1-c)-\theta(1-c)]^3+\ldots$$
$$=\frac{1}{1-[(1-c)-\theta(1-c)]}=\frac{1}{c+\theta(1-c)}. \quad (156)$$

5.1.3 Money demand

Nominal money demand:

$$M^d := P \times L(\underbrace{Y}_{1}, \underbrace{R}_{2}) = P \times L(\underbrace{Y}_{+}, \underbrace{R}_{-}) \tag{157}$$

1. Transaction motive:

$$Y \uparrow \quad \Rightarrow \quad M^d \uparrow . \tag{158}$$

2. Speculative motive (opportunity cost):

$$R \uparrow \quad \Rightarrow \quad M^d \downarrow . \tag{159}$$

Real money demand:

$$\frac{M^d}{P} := L(\underbrace{Y}_{+}, \underbrace{R}_{-}). \tag{160}$$

The money supply, M^s, of an economy is controlled by its central bank. The equilibrium condition in the money market is:

$$M = M^s = M^d \quad \Leftrightarrow \quad \underbrace{M}_{\text{nominal supply}} = \underbrace{P \times L(Y, R)}_{\text{nominal demand}} \quad \Leftrightarrow \quad \underbrace{\frac{M}{P}}_{\text{real supply}} = \underbrace{L(Y, R)}_{\text{real demand}} . \tag{161}$$

5.2 Monetary model with flexible prices

The monetary model of exchange rate determination starts by noting that the nominal exchange rate should, as a first approximation, be equal to the ratio of the purchasing powers of the domestic and foreign currencies:

$$S = \frac{\frac{1}{P}}{\frac{1}{P^*}} = \frac{P^*}{P} \qquad \Leftrightarrow \qquad s = -(p - p^*). \tag{162}$$

Note that explaining the nominal exchange rate in this way is equivalent to assuming that absolute purchasing power parity holds:

$$Q = 1 \qquad \Leftrightarrow \qquad q = 0. \tag{163}$$

When the price level is determined in the money market, we have:

$$S = \frac{M^*}{L(Y^*, R^*)} \times \frac{L(Y, R)}{M}. \tag{164}$$

Now consider a specific function of the real money demand:

$$L(Y, R) = Y^a e^{-bR}. \tag{165}$$

The nominal exchange rate now is:

$$S = \frac{M^*}{(Y^*)^a e^{-bR^*}} \times \frac{(Y)^a e^{-bR}}{M}. \tag{166}$$

Taking the logarithm, we have:

$$s = -(m - m^*) + a(y - y^*) - b(R - R^*), \tag{167}$$

where

$$\begin{aligned} s &= \ln(S), \\ m &= \ln(M), \\ y &= \ln(Y). \end{aligned}$$

Determinants of the nominal exchange rate:

- Difference between the domestic and foreign money supplies

$$M\uparrow \quad \Rightarrow \quad P\uparrow \quad \Rightarrow \quad S\downarrow$$

- Difference between the domestic and foreign incomes

$$Y\uparrow \quad \Rightarrow \quad L(Y,R)\uparrow \quad \Rightarrow \quad P\downarrow \quad \Rightarrow \quad S\uparrow$$

- Difference between the domestic and foreign interest rates

$$R\uparrow \quad \Rightarrow \quad L(Y,R)\downarrow \quad \Rightarrow \quad P\uparrow \quad \Rightarrow \quad S\downarrow$$

Note that the variables M, Y, R influence the exchange rate through changes in the price level, P. For this reason, we refer to this model as the **monetary model with flexible prices.**

5.2.1 Expected rate of depreciation

The expected rate of depreciation is:

$$\begin{aligned}\frac{S-S^e}{S^e} &\approx \ln\left(1+\frac{S-S^e}{S^e}\right)\\ &= \ln\left(\frac{S}{S^e}\right)\\ &= \ln(S)-\ln(S^e)\\ &= s-s^e.\end{aligned} \tag{168}$$

5.2.2 Expectations

The uncovered interest parity implies:

$$R - R^* = s - s^e. \tag{169}$$

So today's exchange rate depends on tomorrow's exchange rate:

$$\begin{aligned} s &= -(m - m^*) + a(y - y^*) - b(s - s^e) \\ &= -\frac{1}{1+b}(m - m^*) + \frac{a}{1+b}(y - y^*) + \frac{b}{1+b}s^e. \end{aligned} \tag{170}$$

Determinants of the nominal exchange rate (cont.):

- Expected exchange rate

$$
\begin{aligned}
&S^e \uparrow \quad \Rightarrow \quad \frac{S - S^e}{S^e} \downarrow \quad \Rightarrow \quad (R - R^*) \downarrow \\
&\Rightarrow \quad (L(\cdot,\cdot) - L(\cdot,\cdot)^*) \uparrow \quad \Rightarrow \quad (P - P^*) \downarrow \quad \Rightarrow \quad S \uparrow
\end{aligned}
$$

The determinants of the expected exchange rate in turn influence the current exchange rate:

$$
s = -\frac{1}{1+b}(m - m^*) + \frac{a}{1+b}(y - y^*) + \frac{b}{1+b}s^e, \tag{171}
$$

$$
\begin{aligned}
\Leftrightarrow s = &-\frac{1}{1+b}(m - m^*) + \frac{a}{1+b}(y - y^*) \\
&+ \frac{b}{1+b}\left[-(m^e - m^{*,e}) + a(y^e - y^{*,e}) - b(R^e - R^{*,e})\right].
\end{aligned} \tag{172}
$$

Determinants of the nominal exchange rate (cont.):

- Difference between the expected domestic and foreign money supplies

$$M^e \uparrow \;\Rightarrow\; P^e \uparrow \;\Rightarrow\; S^e \downarrow \;\Rightarrow\; S \downarrow$$

- Difference between the expected domestic and foreign incomes

$$Y^e \uparrow \;\Rightarrow\; L^e(Y,R) \uparrow \;\Rightarrow\; P^e \downarrow \;\Rightarrow\; S^e \uparrow \;\Rightarrow\; S \uparrow$$

- Difference between the expected domestic and foreign interest rates

$$R^e \uparrow \;\Rightarrow\; L^e(Y,R) \downarrow \;\Rightarrow\; P^e \uparrow \;\Rightarrow\; S^e \downarrow \;\Rightarrow\; S \downarrow$$

5.3 The monetary model with fixed prices

The monetary model of exchange rate determination consists of two periods:

- the short run, t_1, and
- the long run, t_2.

The main difference is that in period t_1 prices are fixed (sticky), whereas in period t_2 they are flexible. As a consequence, the equilibrium in period t_1 is obtained through the adjustment of interest rates and that in period t_2 through the adjustment of prices.

Note there there is also an initial period, t_0, in which the equilibrium is obtained through the adjustment of prices. The analysis of period t_0 is thus analogous to that of period t_1.

The equations of the structural model are the following:

$$q_1 = p_1 - p_1^* + s_1, \tag{173}$$
$$m_1 - p_1 = ay_1 - bR_1, \tag{174}$$
$$m_1^* - p_1^* = ay_1^* - bR_1^*, \tag{175}$$
$$R_1 = R_1^* + s_1 - s_1^e, \tag{176}$$
$$s_1^e = s_2, \tag{177}$$

$$q_2 = p_2 - p_2^* + s_2, \tag{178}$$
$$m_2 - p_2 = ay_2 - bR_2, \tag{179}$$
$$m_2^* - p_2^* = ay_2^* - bR_2^*, \tag{180}$$
$$R_2 = R_2^* + s_2 - s_2^e \tag{181}$$
$$s_2^e = s_2, \tag{182}$$
$$q_2 = 0. \tag{183}$$

The model's equations represent:

- Definition of the real exchange rate
- Equilibrium in the money market (both at home and abroad)
- Uncovered interest parity
- Rational expectations
- Purchasing power parity (PPP) in the long run, but not in the short run

The reduced form of the model in period t_2 is:

$$p_2 = m_2 - ay + bR, \tag{184}$$

$$p_2^* = m_2^* - ay^* - bR^*, \tag{185}$$

$$R_2 = R_2^*, \tag{186}$$

$$\begin{aligned} s_2 &= -(p_2 - p_2^*) \\ &= -(m_2 - m_2^*) + a(y_2 - y_2^*), \end{aligned} \tag{187}$$

$$\begin{aligned} s_2^e &= s_2 \\ &= -(m_2 - m_2^*) + a(y_2 - y_2^*), \end{aligned} \tag{188}$$

$$q_2 = 0, \tag{189}$$

The reduced form of the model in period t_1 is:

$$\begin{aligned} s_1^e &= s_2 \\ &= -(m_2 - m_2^*) + a(y_2 - y_2^*), \end{aligned} \tag{190}$$

$$R_1 = \frac{1}{b}(-m_1 + p_1 + ay_1), \tag{191}$$

$$R_1^* = \frac{1}{b}(-m_1 + p_1 + ay_1), \tag{192}$$

$$\begin{aligned} s_1 &= R_2 - R_2^* + s_1^e \\ &= \frac{1}{b}\left[-(m_1 - m_1^*) + (p_1 - p_1^*) + a(y_1 - y_1^*)\right] - (m_2 - m_2^*) + a(y - y^*), \end{aligned} \tag{193}$$

$$\begin{aligned} q_1 &= (p_1 - p_1^*) + s_1 \\ &= (p_1 - p_1^*) + \frac{1}{b}\left[-(m_1 - m_1^*) + (p_1 - p_1^*) + a(y - y^*)\right] - (m_2 - m_2^*) + a(y - y^*). \end{aligned} \tag{194}$$

Now we analyse what happens when we increase the domestic money supply:

- $m_0 = m$,
- $m_1 = m_2 = m_0 + \delta$.

We will have to distinguish the equilibria in the three periods:

- the initial equilibrium in period t_0,
- the equilibrium after the change in monetary policy (or any other exogenous change) in period t_1 (an instant after t_0),
- the equilibrium in the long run (t_2).

Regarding the expected nominal exchange rate, s^e, we assume the following:

- $s_0^e = s_0$,
- $s_1^e = s_2$,
- $s_2^e = s_2$.

Before determining the equilibrium in period t_1, we will thus first have to determine the equilibrium in period t_2. But the equilibrium in period t_2 is determined in exactly the same way as in the model with flexible prices:

$$\begin{aligned} s_2 &= s_1^e \\ &= (m_2^* - m_2) - a(y_2^* - y_2) + b(R_2^* - R_2) + q_2 \\ &= (m_0^* - m_0 - \delta) - a(y_0^* - y_0) + b(R_0^* - R_0) + q_0 \\ &= s_0 - \delta \\ &< s_0. \end{aligned} \tag{195}$$

Therefore in period 1 the following happens:

- Increase in the domestic money supply:

$$m_1 = m_0 + \delta. \tag{196}$$

- Equilibrium in the money market:

$$m_1 - p_1 = ay_1 - bR_1, \tag{197}$$

$$\begin{aligned} R_1 &= \frac{1}{b}(-\underbrace{(m_0+\delta)}_{=m_1}+p_0+ay_0) \\ &= R_0-\frac{\delta}{b}. \end{aligned} \tag{198}$$

- Interest rate parity:

$$R_1 = R_1^* + s_1 - s_1^e, \tag{199}$$

$$\begin{aligned} s_1 &= R_1 - R_1^* + s_1^e \\ &= \left(R_0-\frac{\delta}{b}\right) - R_0^* + (s_0-\delta) \\ &= s_0-\delta-\frac{\delta}{b} \\ &= s_2-\frac{\delta}{b} \\ &< s_2. \end{aligned} \tag{200}$$

- Depreciation rate:

$$s_1 - s_1^e = -\frac{\delta}{b}. \tag{201}$$

Intuitively, we expect an appreciation of the domestic money to compensate domestic investors for the low level of the domestic interest rate. If the domestic currency depreciates 10% between t_0 and t_2, it will have to depreciate even more, for example 20%, between t_0 and t_1 in order to be able to appreciate between t_1 and t_2.

We therefore find that in this model there is an **”overshooting” of the exchange rate.** This means that in period t_1 the nominal exchange rate falls below its long-run equilibrium level:

$$s_1 < s_2 < s_0. \tag{202}$$

Observations in accordance with the empirical evidence:

- In the short run, the real exchange rate may diverge from its long-run equilibrium.
- The exchange rate is more volatile than the underlying economic variables (”overshooting”).
- An increase in the money supply is associated with a fall in the interest rate, and vice versa.
- In contrast to the monetary model with flexible prices, a fall in the domestic interest rate is associated with a depreciation of the exchange rate.

6 Portfolio balance model

The portfolio balance model has the following characteristics:

- There are three assets in the economy between which the investors have to choose: money, domestic bonds and foreign bonds.
- Besides looking for high returns on their investments, the investors aim to diversify their portfolio to minimize risks.
- As a consequence, the interest rate parity does not hold exactly anymore. Instead changes in the returns of different assets induce gradual portfolio adjustments.

6.1 The model's equations

A simple version of the model is as follows:

$$\frac{M}{W} = m\left(R, R^* + RD\right), \tag{203}$$

$$\frac{B}{W} = b\left(R, R^* + RD\right), \tag{204}$$

$$\frac{\frac{1}{S}B^*}{W} = b^*\left(R, R^* + RD\right), \tag{205}$$

$$W = M + B + \frac{1}{S}B^*, \tag{206}$$

$$\Delta B^* = CB + R^* B^* \quad (\Delta B^* = CA = -FA), \tag{207}$$

where

$$\begin{aligned} RD &= \text{rate of depreciation}, \\ B &= \text{domestic bonds}, \\ B^* &= \text{foreign bonds}, \\ W &= \text{national wealth}, \\ CB &= \text{commercial balance}, \end{aligned} \tag{208}$$

$$
\begin{aligned}
m_1 &< 0, & b_1 &> 0, & b_1^* &< 0, \\
m_2 &< 0, & b_2 &< 0, & b_2^* &> 0.
\end{aligned}
\tag{209}
$$

We only analyse the model in the short run, in which the variables can be classified in the following way:

- Endogenous variables: S, R, R^*, W, ΔB^*
- Exogenous variables: M, B, B^*, RD, CB

We assume that agents have static expectations; this means that RD is constant.

6.2 Equilibrium in the short run

The equations of the model provide us with three curves relating S (vertical axis) with R (horizontal axis):

Curve	Equation	Relationship between S and R	Slope
MM	$\frac{M}{W} = m\left(R, R^* + RD\right)$	$S \uparrow \Rightarrow \frac{M}{W} \uparrow \Rightarrow R \downarrow$	negative
BB	$\frac{B}{W} = b\left(R, R^* + RD\right)$	$S \uparrow \Rightarrow \frac{B}{W} \uparrow \Rightarrow R \uparrow$	positive
BB^*	$\frac{SB^*}{W} = b^*\left(R, R^* + RD\right)$	$S \uparrow \Rightarrow \frac{\frac{1}{S}B^*}{W} \downarrow \Rightarrow R \uparrow$	positive (less than BB)

6.3 Changes in the short run equilibrium

We analyse three changes:

- Increase in the money supply produced by the purchase of domestic bonds:

$$
\begin{aligned}
M \uparrow \quad &\Rightarrow \quad MM \leftarrow \\
B \downarrow \quad &\Rightarrow \quad BB \leftarrow
\end{aligned}
$$

  New equilibrium: $R \downarrow$, $S \downarrow$.

  The increase in the money supply implies a depreciation of the domestic currency.

- Increase in the money supply produced by the purchase of foreign bonds:

$$
\begin{aligned}
M \uparrow \quad &\Rightarrow \quad MM \leftarrow \\
B^* \downarrow \quad &\Rightarrow \quad BB^* \rightarrow
\end{aligned}
$$

  New equilibrium: $R \downarrow$, $S \downarrow$.

  The increase in the money supply implies a depreciation of the domestic currency.

- Increase in the national wealth produced by a rise in the supply of foreign bonds (= current account surplus):

$$
\begin{aligned}
B^* \uparrow \quad &\Rightarrow \quad \frac{M}{W} \downarrow \quad \Rightarrow \quad MM \rightarrow \\
B^* \uparrow \quad &\Rightarrow \quad \frac{B}{W} \downarrow \quad \Rightarrow \quad BB \leftarrow \\
B^* \uparrow \quad &\Rightarrow \quad \frac{SB^*}{W} \uparrow \quad \Rightarrow \quad BB^* \leftarrow
\end{aligned}
$$

New equilibrium: R const., $S \uparrow$.

A surplus on current account is associated with an appreciation of the domestic currency, a deficit with a depreciation.

7 Empirical evidence on traditional exchange rate models

During and after the Second World War, the Bretton Woods system was established, which foresaw fixed exchange rates between the most important currencies. However, in the early 1970s, this system collapsed and the countries began to let their currencies float against each other.

A couple of years later, Meese and Rogoff (1983) used the newly available time series of floating exchange rates to test the validity of several popular macroeconomic models of the time. They analysed for instance the monetary models of exchange rate determination, both with flexible and with fixed prices. They showed that these models were not delivering better forecasts than a simple random walk model, putting in doubt the usefulness of the models they considered.

As other studies showed, it has neither been possible to confirm the empirical validity of the portfolio balance model.

Notwithstanding these negative results, it must be stressed that money growth, inflation and the depreciation of the domestic currency are highly correlated in the long run (or in the short run when money growth is very strong, as it is the case, for example, during hyperinflations).

8 The currency flow model

8.1 Money flows and the exchange rate

Nominal exchange rate:

- Suppose that, as in the monetary model, the fundamental level of the nominal exchange rate is given by the **ratio of the purchasing powers** of the domestic and foreign currencies, respectively. In logarithms, the purchasing power of the domestic currency is $-p^{\mathrm{H}}$ and that of the foreign currency $-p^{\mathrm{F}}$.
- In addition, assume that the forces of supply and demand in the foreign exchange market can drive the exchange rate above or below its fundamental level. These forces are measured by a variable called **currency market pressure** and denoted as m^{HF}.
- Hence the nominal exchange rate is given by:

$$s = -(p^{\mathrm{H}} - p^{\mathrm{F}}) + \xi m^{\mathrm{HF}}. \tag{210}$$

Real exchange rate:

- The real exchange rate is thus **solely driven by currency market pressure:**

$$
\begin{aligned}
q &= s + p^{\mathrm{H}} - p^{\mathrm{F}} \\
&= -(p^{\mathrm{H}} - p^{\mathrm{F}}) + \xi m^{\mathrm{HF}} + p^{\mathrm{H}} - p^{\mathrm{F}} \\
&= \xi m^{\mathrm{HF}}.
\end{aligned} \tag{211}
$$

- Note that a change in the **domestic and foreign price levels** does **not affect the real exchange rate:**
  - A rise in p^{H}, for instance, makes the ratio of domestic to foreign prices, $p^{\mathrm{H}} - p^{\mathrm{F}}$, rise.
  - At the same time, however, a rise in p^{H} reduces the purchasing power of the domestic money, implying a nominal depreciation, or a fall in s.
  - According to the definition of the real exchange rate, both effects exactly offset each other, leaving the real exchange rate unchanged.

- **Fixing the nominal exchange rate** implies setting $m^{\text{HF}} = \frac{1}{\xi}(\bar{s} + p^{\text{H}} - p^{\text{F}})$, where $\bar{s}$ is a constant. This could happen, for example, through reserve intervention. In this case, the real exchange rate does again depend on the domestic and foreign price levels:

$$\begin{aligned} q &= s + p^{\text{H}} - p^{\text{F}} \\ &= -(p^{\text{H}} - p^{\text{F}}) + \xi m^{\text{HF}} + p^{\text{H}} - p^{\text{F}} \\ &= -(p^{\text{H}} - p^{\text{F}}) + (\bar{s} + p^{\text{H}} - p^{\text{F}}) + p^{\text{H}} - p^{\text{F}} \\ &= \bar{s} + p^{\text{H}} - p^{\text{F}}. \end{aligned} \tag{212}$$

8.2 Money flows in the balance of payments

Let us define the net foreign asset position (NFA), or net international investment position (IIP), and its components:

Net foreign asset position			
1.	Net foreign asset position	z^{HF}	
Components			
2.	Net foreign equity holdings	e^{HF}	
3.	Net foreign bond holdings	b^{HF}	
4.	Net foreign money holdings	m^{HF}	
5.	Net official reserve holdings	$b^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	$(= b^{\bar{\mathrm{H}}\mathrm{F}} - b^{\bar{\mathrm{F}}\mathrm{H}})$

We thus obtain the following classification of the balance of payments:

Current account		
1. Current account	Δz^{HF}	$(= CA + KA)$
Financial account		
2. Net foreign equity acquisitions (= equity investment ”outflows”)	Δe^{HF}	
3. Net foreign bond acquisitions (= bond investment ”outflows”)	Δb^{HF}	
4. Net money inflows	Δm^{HF}	
5. Net official reserve inflows	$\Delta b^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	$(= \Delta b^{\bar{\mathrm{H}}\mathrm{F}} - \Delta b^{\bar{\mathrm{F}}\mathrm{H}})$

Balance of payments identity:

$$\Delta z^{\mathrm{HF}} = \Delta e^{\mathrm{HF}} + \Delta b^{\mathrm{HF}} + \Delta m^{\mathrm{HF}} + \Delta b^{\bar{\mathrm{H}}\bar{\mathrm{F}}}. \tag{213}$$

Net money inflows under **flexible exchange rate:**

$$\Rightarrow \quad \Delta m^{\mathrm{HF}} = \Delta z^{\mathrm{HF}} - (\Delta e^{\mathrm{HF}} + \Delta b^{\mathrm{HF}}). \tag{214}$$

Net official reserve changes under **fixed real exchange rate:**

$$\Rightarrow \quad \Delta b^{\bar{\mathrm{H}}\bar{\mathrm{F}}} = \Delta z^{\mathrm{HF}} - (\Delta e^{\mathrm{HF}} + \Delta b^{\mathrm{HF}}). \tag{215}$$

Net money inflows and net official reserve changes under **managed float:**

$$\begin{aligned} \Rightarrow \quad \Delta m^{\mathrm{HF}} + \Delta b^{\bar{\mathrm{H}}\bar{\mathrm{F}}} &= \Delta z^{\mathrm{HF}} - (\Delta e^{\mathrm{HF}} + \Delta b^{\mathrm{HF}}) \\ &= \Delta x^{\mathrm{HF}}. \end{aligned} \tag{216}$$

The variable Δx^{HF} is what is called the **”international cash flow”** of the country in question.

8.3 Typical movements of the balance of payments and the exchange rate

We will analyse some typical situations and their effect on the behaviour of the exchange rate.

All the examples share the following assumptions:

- Unless otherwise stated, prices are constant and $p^{\mathrm{H}} = p^{\mathrm{F}}$, implying that the nominal and real exchange rates coincide (an exception from this assumption is made in case 8).
- The balance of payments identity holds in every moment:
$$\Delta z^{\mathrm{HF}} = \Delta e^{\mathrm{HF}} + \Delta b^{\mathrm{HF}} + \Delta m^{\mathrm{HF}} + \Delta b^{\bar{\mathrm{H}}\bar{\mathrm{F}}}. \tag{217}$$
- The nominal exchange rate is given by:
$$s = -(p^{\mathrm{H}} - p^{\mathrm{F}}) + \xi m^{\mathrm{HF}}. \tag{218}$$
- The parameter ξ equals 0.01.
- The nominal exchange rate, S_t, is initially equal to one (you buy, for example, a euro with one dollar):
$$S_0 = 1.00 \qquad \Leftrightarrow \qquad s_0 = 0.00. \tag{219}$$
- The real exchange rate is given by:
$$q = s + p^{\mathrm{H}} - p^{\mathrm{F}}. \tag{220}$$

8.3.1 Case 1: Currency exchange

A Spanish resident on holidays in the USA exchanges 100 euros for 100 dollars.

Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	Δs_t	s_t
1	Pay 100€				-100€			
1	Receive 100\$				$+100$€			
1	Balance	0€	0€	0€	0€	0€	0.00	0.00

8.3.2 Case 2: Barter

A Spanish resident exchanges a good with a value of 100 euros for another good of the same value with a foreigner.

Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	Δs_t	s_t
1	Give a good	$+100$						
1	Receive a good	-100						
1	Balance	0	0	0	0	0	$0,00$	$0,00$

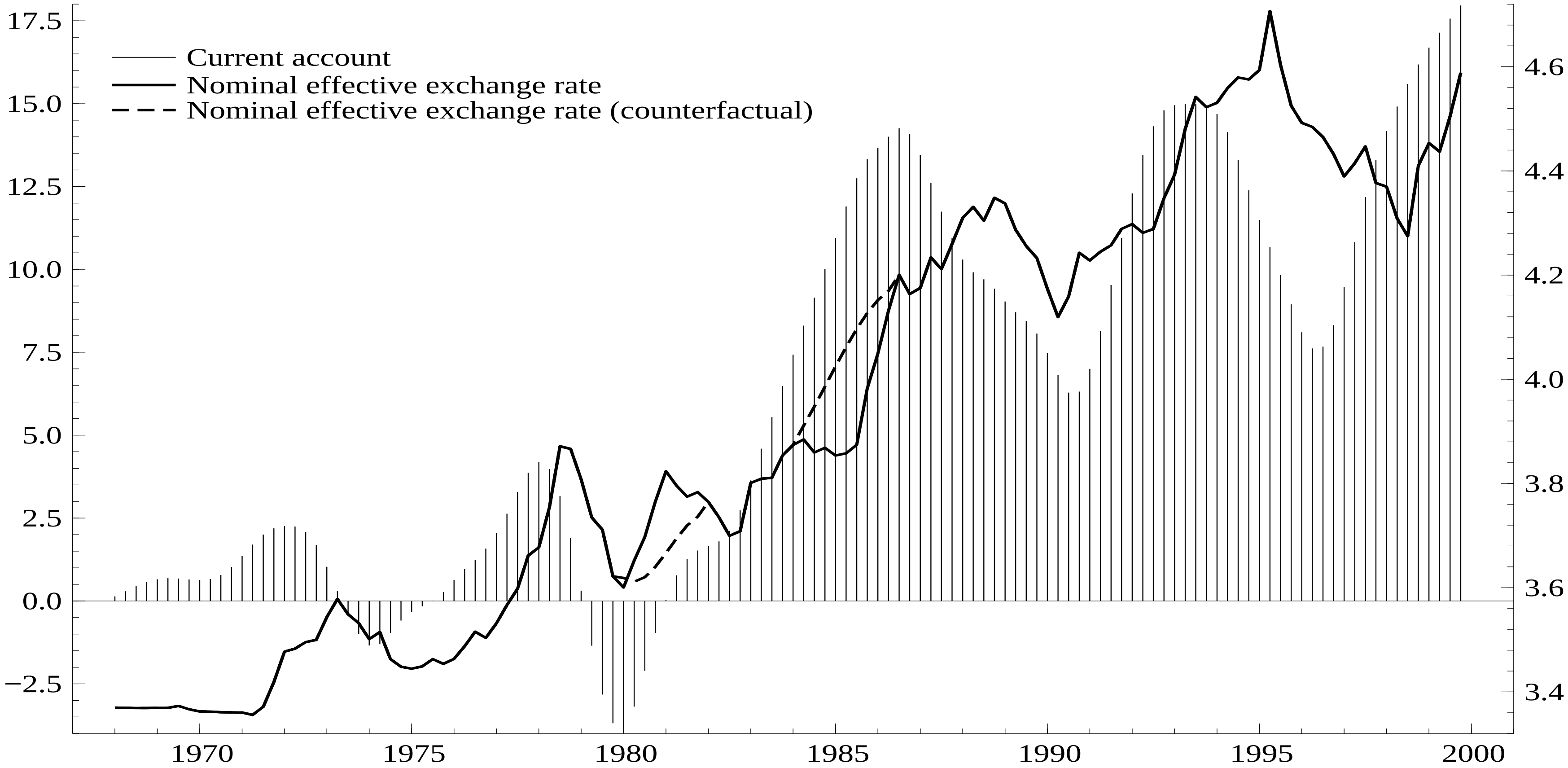
8.3.3 Case 3: Current account transactions paid with money

A Spanish resident exports goods abroad with value of 100 euros.

Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	Δs_t	s_t
1	Export	$+100$						
1	Money payment				$+100$			
1	Balance	$+100$	0	0	$+100$	0	1.00	1.00

Example of Japan:

- country with the largest current account surplus and the largest fluctuations of the current account balance (during the 1980s, 1990s and 2000s);
- strong correlation between the level of the current account and the changes in the exchange rate.



Japanese current account and counterfactual exchange rate.

8.3.4 Case 4: Current account transactions financed by a temporary loan (adaptive capital flows)

A Spanish resident exports a good with a value of 100 euros and provides a trade credit of the same amount to the foreign importer, to be paid back after a year.

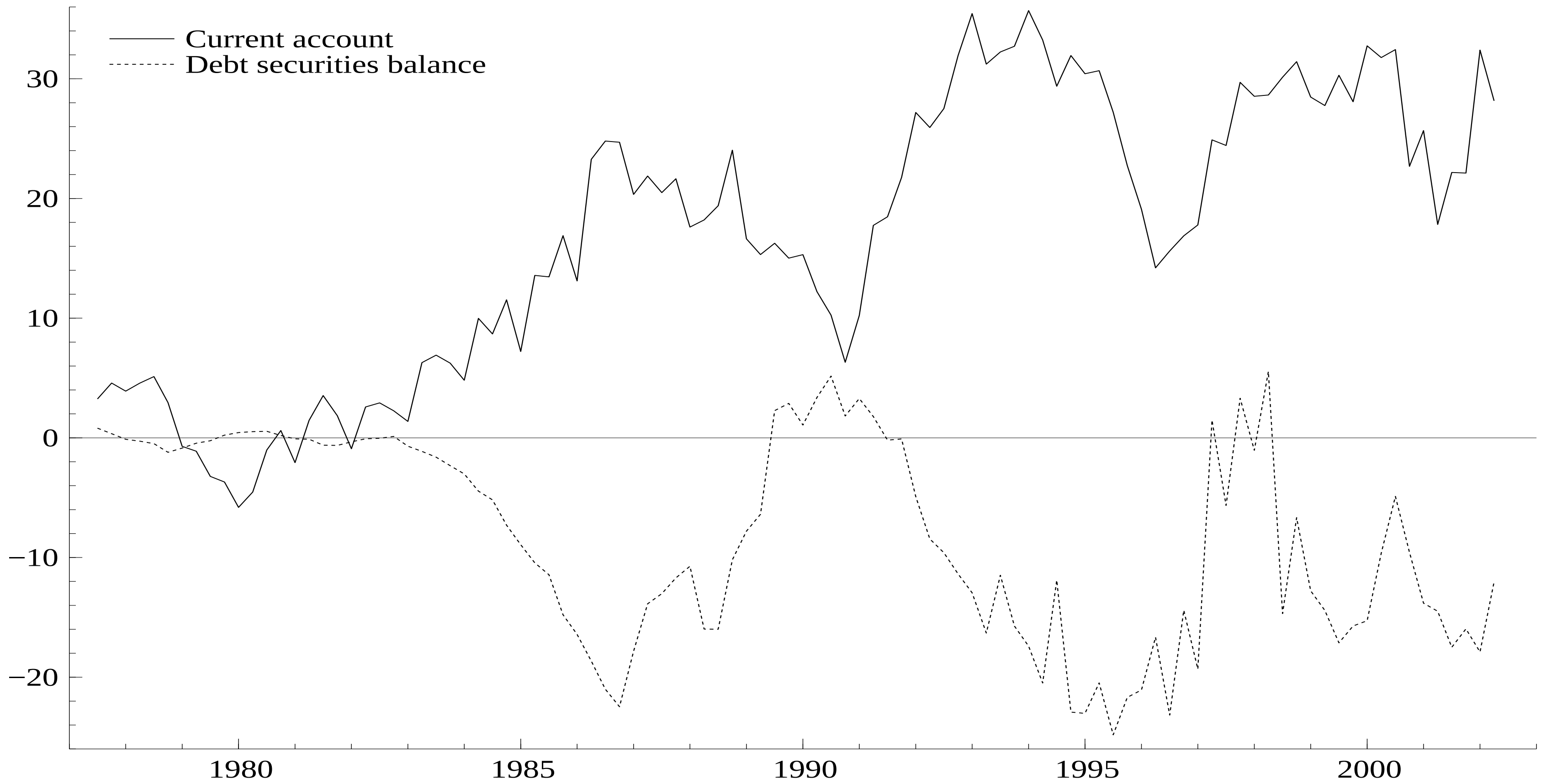
Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	Δs_t	s_t
1	Export	$+100$						
1	Loan			$+100$				
1	Balance	$+100$	0	$+100$	0	0	0.00	0.00
2	Loan repayment			-100				
2	Money payment				$+100$			
2	Balance	0	0	-100	$+100$	0	1.00	1.00

Conclusion:

- In the presence of international loans, the effect of an imbalance in the current account on the exchange rate of a country may be delayed.

Example of Japan:

- Japan liberalized its financial account in the late 1970s and the first half of the 1980s.
- It is observed that the duration of the cyclical movements of the Japan's current account and of its exchange rate was getting longer.



Current account and lending in Japan.

8.3.5 Case 5: The effect of the real exchange rate on the commercial balance

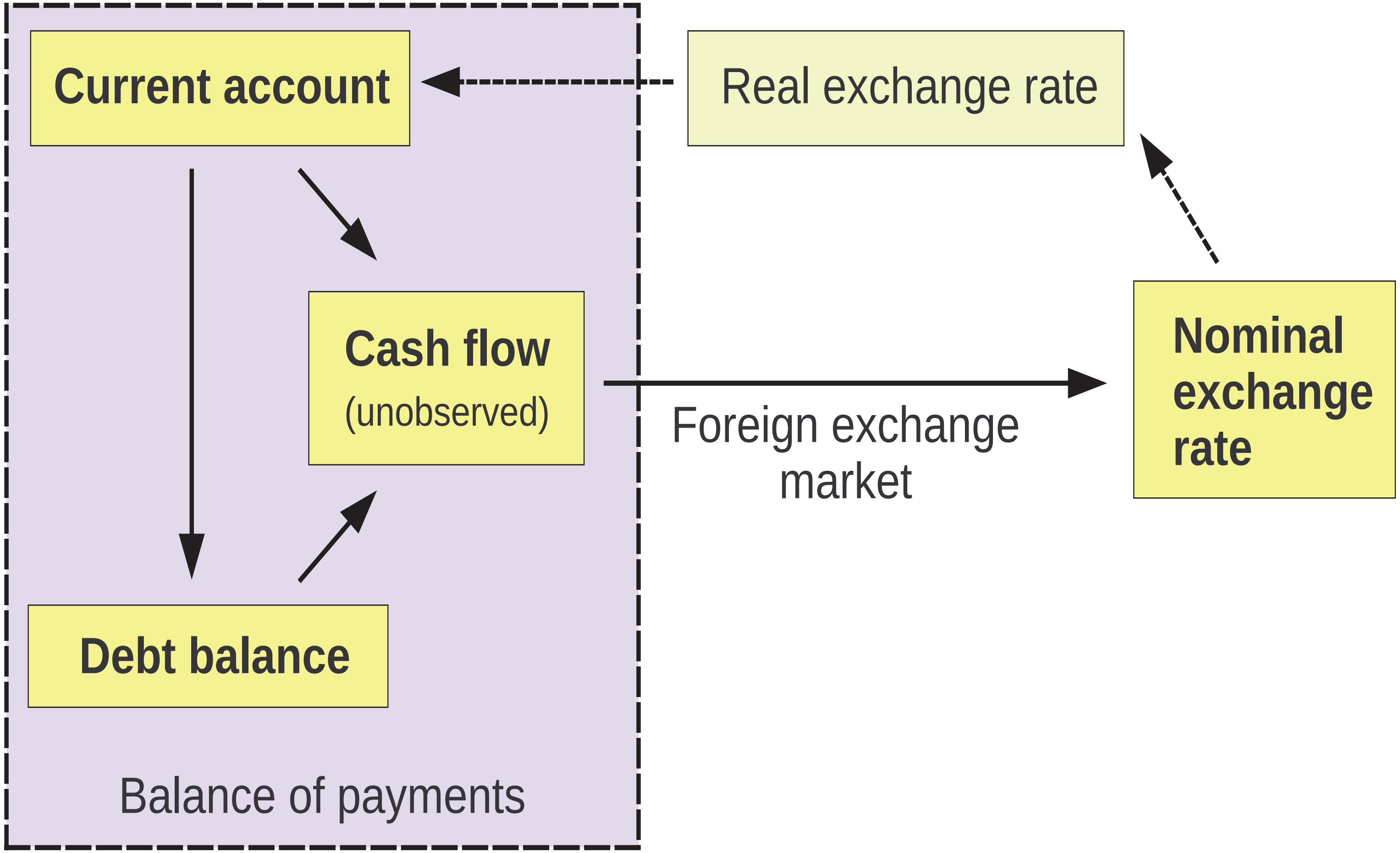
- Same as case 3, but now we allow the real exchange rate to influence the current account (through its effect on the commercial balance): $q_t \to \Delta z_t^{\mathrm{HF}}$.
- Note that before we were only considering the effect of the current account on the currency flows and the nominal exchange rate: $\Delta z_t^{\mathrm{HF}} \to m_t^{\mathrm{HF}} \to s_t$.
- More specifically, we assume that the current account moves according to the following equation:

$$\Delta z_t^{\mathrm{HF}} = \Delta z_{t-1}^{\mathrm{HF}} - 100\, q_{t-1}. \tag{221}$$

Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	Δs_t	s_t
1	Balance	$+100$	0	0	$+100$	0	$+1.00$	$+1.00$
2	Balance	0	0	0	0	0	0.00	$+1.00$
3	Balance	-100	0	0	-100	0	-1.00	0.00
4	Balance	-100	0	0	-100	0	-1.00	-1.00
5	Balance	0	0	0	0	0	0.00	-1.00
6	Balance	$+100$	0	0	$+100$	0	$+1.00$	0.00
7	Balance	$+100$	0	0	$+100$	0	$+1.00$	$+1.00$
8	Balance	0	0	0	0	0	0.00	$+1.00$

Conclusion:

- The interaction between the current account and the exchange rate (nominal and real) produces cycles in both variables.
- These cycles are also observed in reality, for example in Japan, United States and Korea.



Cash flow and exchange rate determination.

8.3.6 Case 6: Autonomous capital flows

- In case 4, we saw what happens when a current account surplus (o deficit) is financed by international loans; in this case, the capital flows were adaptive since they geared towards the external financing needs associated with the current account imbalance.
- Today, with liberalized financial markets in many countries, capital flows often move independently and do not necessarily reflect the movements of the current account (”autonomous capital flows”).

We now consider the case of foreign investors who invest in our country who lend money to us, which has to be returned within one year.

Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	Δs_t	s_t
1	Loan provision			-100				
1	Money payment				$+100$			
1	Balance	0	0	-100	$+100$	0	$+1,00$	$+1,00$
2	Loan amortization			$+100$				
2	Money payment				-100			
2	Balance	0	0	$+100$	-100	0	$-1,00$	$0,00$

8.3.7 Case 7: Currency crises due to autonomous capital inflows and an appreciation of the real exchange rate

Capital flows received by Asia (in billions of US dollars):

Period	Value (per year)
1977–1982	15.8
1983–1989	16.7
1990–1994	40.1
1995	95.8
1996	110.4
1997	13.9

Capital flows received by emerging markets (in billions of US dollars):

Period	Value (per year)
1977–1982	30.5
1983–1989	8.8
1990–1994	120.8
1995	192.0
1996	240.8
1997	173.7

We now consider the possible effects of autonomous capital flows during various periods.

- As before, we assume that the real exchange rate affects the current account as follows:

$$\Delta z_t^{\mathrm{HF}} = \Delta z_{t-1}^{\mathrm{HF}} - 100\, q_{t-1}. \tag{222}$$

	Δz_t^{HF}	Δe_t^{HF}	Δm_t^{HF}	m_t^{HF}	s_t	q_t
		exog.				
0	0			0		
1	0	-100	100	100	1	1
2	-100	-200	100	200	2	2
3	-300	-300	0	200	2	2
4	-500	600	-1100	-900	-9	-9
5	400	0	400	-500	-5	-5
6	900	0	900	400	4	4

8.3.8 Case 8: Currency crises due to persistent inflation

We now examine how persistent inflation can lead to the appreciation of the real exchange rate, the deterioration of the current account balance and eventually to the breakdown of the fixed exchange rate regime.

- As before, we assume that the real exchange rate affects the current account as follows:

$$\Delta z_t^{\mathrm{HF}} = \Delta z_{t-1}^{\mathrm{HF}} - 100\, q_{t-1}. \tag{223}$$

- As long as the domestic central bank has sufficient official reserves, they are used to keep the nominal exchange rate stable:

$$\Delta b^{\bar{\mathrm{H}}\bar{\mathrm{F}}} = \Delta z^{\mathrm{HF}} - \Delta m_t^{\mathrm{HF}} = \Delta z^{\mathrm{HF}} - \frac{1}{\xi}(\Delta p^{\mathrm{H}} - \Delta p^{\mathrm{F}}). \tag{224}$$

- When official reserves are exhausted, the exchange rate peg is given up:

$$\Delta m^{\mathrm{HF}} = \Delta z^{\mathrm{HF}}. \tag{225}$$

	Δz_t^{HF}	Δm_t^{HF}	m_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	$b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	s_t	p^{H}	p^{F}	$p^{\mathrm{H}} - p^{\mathrm{F}}$	q_t
							exog.	exog.		
0	0		0		2500	0				0
1	0	0	0	0	2500	0	0	0	0	0
2	0	100	100	-100	2400	0	1	0	1	1
3	-100	100	200	-200	2200	0	2	0	2	2
4	-300	100	300	-400	1800	0	3	0	3	3
5	-600	100	400	-700	1100	0	4	0	4	4
6	-1000	100	500	-1100	0	0	5	0	5	5
7	-1500	-1500	-1000	0	0	-16	6	0	6	-10
8	-500	-500	-1500	0	0	-22	7	0	7	-15

8.3.9 Case 9: Currency crises due to boom-and-bust cycles

We now examine how boom-and-bust cycles can lead to currency crises:

- The current account results from the national income identity:

$$\Delta z_t = Y_t^{\mathrm{H}} - C_t^{\mathrm{H}} - \Delta K_t^{\mathrm{H}}. \tag{226}$$

- For simplicity, consumption equals income:

$$C_t^{\mathrm{H}} = Y_t^{\mathrm{H}}. \tag{227}$$

- Real investment equals the difference between the desired and the previous capital stock. However, since changes in the capital stock imply large sunk costs, investors limit themselves to a maximum investment of 1000 and a maximum disinvestment of 500 per period. Therefore we have:

$$\Delta K_t^{\mathrm{H}} = \max\left(\min\left(K_t^{\mathrm{H},d} - K_{t-1}^{\mathrm{H}}, 1000\right), -500\right). \tag{228}$$

- To keep things simple, desired net foreign investments in the home country equal the desired capital stock:

$$e_t^{\mathrm{FH},d} = -e_t^{\mathrm{HF},d} = K_t^{\mathrm{H},d}. \tag{229}$$

- Net foreign equity investment equals the difference between the desired and the previous stock of net cumulative foreign investment:

$$\Delta e_t^{\mathrm{HF}} = e_t^{\mathrm{HF},d} - e_{t-1}^{\mathrm{HF}}. \tag{230}$$

- As long as the domestic central bank has sufficient official reserves, they are used to keep the nominal exchange rate stable:

$$\Delta b^{\bar{\mathrm{H}}\bar{\mathrm{F}}} = \Delta z^{\mathrm{HF}} - \Delta e^{\mathrm{HF}}. \tag{231}$$

- When official reserves are exhausted, the exchange rate peg is given up:

$$\Delta m^{\mathrm{HF}} = \Delta z^{\mathrm{HF}} - \Delta e^{\mathrm{HF}}. \tag{232}$$

	Y_t^{H}	C_t^{H}	$K_t^{\mathrm{H},d}$	ΔK_t^{H}	K_t^{H}	Δz_t^{HF}	$e_t^{\mathrm{HF},d}$	Δe_t^{HF}	e_t^{HF}	Δm_t^{HF}	m_t^{HF}	$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	$b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	s_t
	exog.		exog.				exog.							
0					5000				-5000		0		2000	0
1	1000	1000	5000	0	5000	0	-5000	0	-5000	0	0	0	2000	0
2	1000	1000	5000	0	5000	0	-5000	0	-5000	0	0	0	2000	0
3	1000	1000	5000	0	5000	0	-5000	0	-5000	0	0	0	2000	0
4	1000	1000	7500	1000	6000	-1000	-7500	-2500	-7500	0	0	1500	3500	0
5	1000	1000	10000	1000	7000	-1000	-10000	-2500	-10000	0	0	1500	5000	0
6	1000	1000	10000	1000	8000	-1000	-10000	0	-10000	0	0	-1000	4000	0
7	1000	1000	10000	1000	9000	-1000	-10000	0	-10000	0	0	-1000	3000	0
8	1000	1000	10000	1000	10000	-1000	-10000	0	-10000	0	0	-1000	2000	0
9	1000	1000	7500	-500	9500	500	-7500	2500	-7500	0	0	-2000	0	0
10	1000	1000	5000	-500	9000	500	-5000	2500	-5000	-2000	-2000	0	0	-20
11	1000	1000	5000	-500	8500	500	-5000	0	-5000	0	-2000	500	500	-20
12	1000	1000	5000	-500	8000	500	-5000	0	-5000	0	-2000	500	1000	-20

Summary on cases 7, 8 and 9: Three explanations of currency crises

- In case 7, we have seen that large autonomous capital inflows into a country can cause an appreciation of the national currency and a worsening of the current account:

$$e_t^{\mathrm{HF}} \downarrow \quad \Rightarrow \quad s_t \uparrow \quad \Rightarrow \quad q_t \uparrow \quad \Rightarrow \quad \Delta z_t^{\mathrm{HF}} \downarrow . \tag{233}$$

- In case 8, we have seen that if inflation is persistently higher at home than abroad, pegging the nominal exchange rate leads to a real appreciation and a deterioration of the current account:

$$s_t \text{ const., } p_t^{\mathrm{H}} > p_t^{\mathrm{F}} \quad \Rightarrow \quad q_t \uparrow \quad \Rightarrow \quad \Delta z_t^{\mathrm{HF}} \downarrow . \tag{234}$$

- In case 9, we have seen that economic booms tend to stimulate real investment (and consumption) and foreign capital inflows. Since real investment is associated with large sunk costs, real investment does not fluctuate as much as net flows of foreign capital (which consist only of financial, rather than real, investments):
  - Period t:

$$\Delta K_t^{\mathrm{H}} > 0 \quad \Rightarrow \quad \Delta z_t^{\mathrm{HF}} < 0, \tag{235}$$

$$\Delta e_t^{\mathrm{HF}} \ll 0, \tag{236}$$

$$\Delta z_t^{\mathrm{HF}} - \Delta e_t^{\mathrm{HF}} > 0 \quad \Rightarrow \quad \Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}} > 0 \quad \Rightarrow \quad s_t \text{ const.} \tag{237}$$

  - Period $t+1$:

$$\Delta K_{t+1}^{\mathrm{H}} > 0 \quad \Rightarrow \quad \Delta z_{t+1}^{\mathrm{HF}} < 0, \tag{238}$$

$$\Delta e_{t+1}^{\mathrm{HF}} = 0, \tag{239}$$

$$\Delta z_{t+1}^{\mathrm{HF}} - \Delta e_{t+1}^{\mathrm{HF}} < 0 \quad \Rightarrow \quad \Delta b_{t+1}^{\bar{\mathrm{H}}\bar{\mathrm{F}}} < 0 \quad \Rightarrow \quad s_{t+1} \text{ const.} \tag{240}$$

  - Period $t+2$:

$$\Delta K_{t+2}^{\mathrm{H}} < 0 \quad \Rightarrow \quad \Delta z_{t+2}^{\mathrm{HF}} > 0, \tag{241}$$

$$\Delta e_{t+2}^{\mathrm{HF}} \gg 0, \tag{242}$$

$$\Delta z_{t+2}^{\mathrm{HF}} - \Delta e_{t+2}^{\mathrm{HF}} < 0 \quad \Rightarrow \quad \text{first } \Delta b_{t+2}^{\bar{\mathrm{H}}\bar{\mathrm{F}}} < 0, \text{ then } \Delta m_{t+2}^{\mathrm{HF}} < 0 \quad \Rightarrow \quad s_{t+2} \downarrow . \tag{243}$$

8.3.10 Case 10: Official intervention in the foreign exchange market

There are two possibilities:

- The central bank in our country buys foreign reserves (foreign bonds) to devalue its own currency.

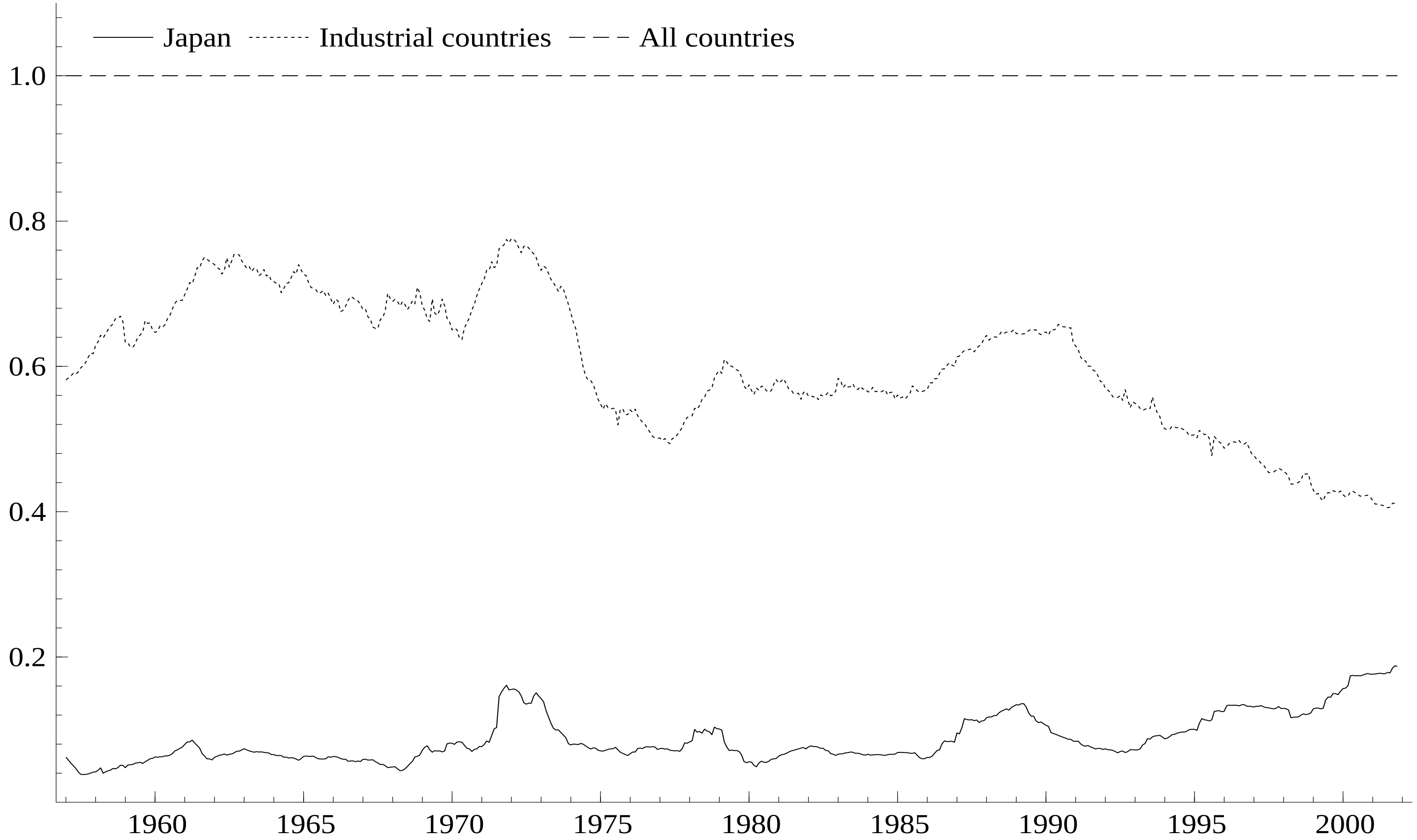
Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\text{H}}\bar{\text{F}}}$	Δs_t	s_t
1	Purchase of reserves					$+100$		
1	Payment of money				-100			
1	Balance	0	0	0	-100	$+100$	-1.00	-1.00

- The central bank in our country sells foreign reserves in order to revaluate its own currency.

Period	Transaction	Δz_t^{HF}	Δe_t^{HF}	Δb_t^{HF}	Δm_t^{HF}	$\Delta b_t^{\bar{\text{H}}\bar{\text{F}}}$	Δs_t	s_t
1	Sale of reserves					-100		
1	Payment of money				$+100$			
1	Balance	0	0	0	$+100$	-100	$+1.00$	$+1.00$

Observations:

- In recent decades, the value of official reserves in the entire world has grown continuously.
- In the past three decades, Japan, China and other Asian economies have accumulated large currency reserves of (both in absolute and relative terms) in an attempt to moderate the pressure on their currencies.
- Since 1990, developing countries have acquired an increasing fraction of world-wide official reserves in an attempt to insulate their currencies from the increasingly volatile international capital flows.



Japan's share of world reserves.

8.4 Intervention in foreign exchange markets

8.4.1 Internal credit and reserves

Balance sheet of the central bank:

Assets	Liabilities
Bonds ($b_t^{\bar{\mathrm{H}}\mathrm{H}}$)	Currency in circulation
Foreign reserves ($b_t^{\bar{\mathrm{H}}\mathrm{F}}$)	Bank deposits

$$
\begin{aligned}
M_t^{\bar{\mathrm{H}}} &= \text{currency} + \text{bank deposits} \\
&= m_t^{\mathrm{H}\bar{\mathrm{H}}} + m_t^{\mathrm{F}\bar{\mathrm{H}}} \\
&= b_t^{\bar{\mathrm{H}}\mathrm{H}} + b_t^{\bar{\mathrm{H}}\mathrm{F}},
\end{aligned} \tag{244}
$$

where

$$
\begin{aligned}
b_t^{\bar{\mathrm{H}}\mathrm{H}} &= \text{internal credit}, \\
b_t^{\bar{\mathrm{H}}\mathrm{F}} &= \text{official reserves}.
\end{aligned}
$$

Let a hat above a variable indicate the growth rate of that variable in discrete time:

$$\hat{x}_t = \frac{\Delta x_t}{x_{t-1}}. \tag{245}$$

Growth rate of money:

$$\hat{M}_t^{\bar{\mathrm{H}}} = \frac{b_t^{\bar{\mathrm{H}}\mathrm{H}}}{M_t^{\bar{\mathrm{H}}}}\,\hat{b}_t^{\bar{\mathrm{H}}\mathrm{H}} + \frac{b_t^{\bar{\mathrm{H}}\mathrm{F}}}{M_t^{\bar{\mathrm{H}}}}\,\hat{b}_t^{\bar{\mathrm{H}}\mathrm{F}}. \tag{246}$$

The central bank creates money:

- by buying domestic bonds ($b_t^{\bar{\mathrm{H}}\mathrm{H}} \uparrow$),
- by buying official reserves ($b_t^{\bar{\mathrm{H}}\mathrm{F}} \uparrow$).

The monetary model can thus be written as follows:

$$s_t = -\ln(b_t^{\bar{\mathrm{H}}\mathrm{H}} + b_t^{\bar{\mathrm{H}}\mathrm{F}}) + \ln(b_t^{\bar{\mathrm{F}}\mathrm{F}} + b_t^{\bar{\mathrm{F}}\mathrm{H}}) + \alpha(y_t^{\mathrm{H}} - y_t^{\mathrm{F}}) - \beta(R_t^{\mathrm{H}} - R_t^{\mathrm{F}}) + \xi m_t^{\mathrm{HF}}. \tag{247}$$

For given levels of all other variables, both the increase of domestic credit (purchase of domestic bonds) and of official reserves (purchase of foreign currency) induce currency depreciations ($s \downarrow$).

The equation can also be written in terms of percentage changes:

$$\begin{aligned}\Delta s_t = &- \left(\frac{b_t^{\bar{\mathrm{H}}\mathrm{H}}}{M^{\bar{\mathrm{H}}}}\,\hat{b}_t^{\bar{\mathrm{H}}\mathrm{H}} - \frac{b_t^{\bar{\mathrm{F}}\mathrm{F}}}{M^{\bar{\mathrm{F}}}}\,\hat{b}_t^{\bar{\mathrm{F}}\mathrm{F}}\right) - \left(\frac{b_t^{\bar{\mathrm{H}}\mathrm{F}}}{M^{\bar{\mathrm{H}}}}\,\hat{b}_t^{\bar{\mathrm{H}}\mathrm{F}} - \frac{b_t^{\bar{\mathrm{F}}\mathrm{H}}}{M^{\bar{\mathrm{F}}}}\,\hat{b}_t^{\bar{\mathrm{F}}\mathrm{H}}\right) \\ &+ \alpha(\Delta y_t^{\mathrm{H}} - \Delta y_t^{\mathrm{F}}) - \beta(\Delta R_t^{\mathrm{H}} - \Delta R_t^{\mathrm{F}}) + \xi \Delta m_t^{\mathrm{HF}}.\end{aligned} \tag{248}$$

8.4.2 Fixed exchange rates

Many countries want to maintain a stable value of their currency. To achieve this, they try to peg their currency to some other currency like the US dollar or the euro.

The advantages of a fixed exchange rate include, for example:

- Stable demand for, and value of, exports and imports
- Attraction of foreign investments
- Control of inflation
- Control of external debt (denominated in foreign currency)

Control of inflation:

- From the definition of the real exchange rate, we know that:

  $$\Delta s = \pi^{\mathrm{F}} - \pi^{\mathrm{H}} + \Delta q, \tag{249}$$

  where

  $$\pi = \Delta p. \tag{250}$$

- When the relative PPP holds (more likely in the long run):

$$\Delta s = \pi^{\mathrm{F}} - \pi^{\mathrm{H}}. \tag{251}$$

- According to the last equation, a sustained depreciation of the national currency is associated with a higher rate of inflation in our country than abroad.

In fact, there are several regimes. The following is a classification used by the International Monetary Fund (IMF):

- Exchange rate regimes without a national currency as legal tender
- Currency board arrangements
- Conventional fixed exchange rate regimes
- Fixed exchange rates within horizontal bands
- Crawling pegs
- Exchange rates within fluctuation bands
- Managed floats without a preannounced path for the exchange rate
- Independent floating

8.5 Currency crises

8.5.1 Definition

- **Currency crisis:** A currency crisis occurs when a currency depreciates or devaluates substantially in a relatively short time, or when speculative fears of a devaluation put downward pressure on a currency in the foreign exchange market, forcing the central bank to rise the interest rates and to sell its reserves.
- **Synonyms and similar terms:** balance of payments crisis, speculative attack, exchange rate crisis, financial crisis.

8.5.2 Causes of currency crises

Let us recall the fundamental equation that determined the nominal exchange rate:

$$\begin{aligned}
\Delta s_t = &-\left(\frac{b_t^{\bar{\mathrm{H}}\mathrm{H}}}{M^{\bar{\mathrm{H}}}}\hat{b}_t^{\bar{\mathrm{H}}\mathrm{H}} - \frac{b_t^{\bar{\mathrm{F}}\mathrm{F}}}{M^{\bar{\mathrm{F}}}}\hat{b}_t^{\bar{\mathrm{F}}\mathrm{F}}\right) - \left(\frac{b_t^{\bar{\mathrm{H}}\mathrm{F}}}{M^{\bar{\mathrm{H}}}}\hat{b}_t^{\bar{\mathrm{H}}\mathrm{F}} - \frac{b_t^{\bar{\mathrm{F}}\mathrm{H}}}{M^{\bar{\mathrm{F}}}}\hat{b}_t^{\bar{\mathrm{F}}\mathrm{H}}\right) \\
&+ \alpha(\Delta y_t^{\mathrm{H}} - \Delta y_t^{\mathrm{F}}) - \beta(\Delta R_t^{\mathrm{H}} - \Delta R_t^{\mathrm{F}}) + \xi \Delta m_t^{\mathrm{HF}} \\
= &-\left(\frac{b_t^{\bar{\mathrm{H}}\mathrm{H}}}{M^{\bar{\mathrm{H}}}}\hat{b}_t^{\bar{\mathrm{H}}\mathrm{H}} - \frac{b_t^{\bar{\mathrm{F}}\mathrm{F}}}{M^{\bar{\mathrm{F}}}}\hat{b}_t^{\bar{\mathrm{F}}\mathrm{F}}\right) - \left(\frac{b_t^{\bar{\mathrm{H}}\mathrm{F}}}{M^{\bar{\mathrm{H}}}}\hat{b}_t^{\bar{\mathrm{H}}\mathrm{F}} - \frac{b_t^{\bar{\mathrm{F}}\mathrm{H}}}{M^{\bar{\mathrm{F}}}}\hat{b}_t^{\bar{\mathrm{F}}\mathrm{H}}\right) \\
&+ \alpha(\Delta y_t^{\mathrm{H}} - \Delta y_t^{\mathrm{F}}) - \beta(\Delta DR_t^e + \Delta\omega_t) \\
&+ \xi(\Delta z_t^{\mathrm{HF}} - \Delta e_t^{\mathrm{HF}} - \Delta b_t^{\mathrm{HF}} - \Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}).
\end{aligned} \tag{252}$$

where

$$\begin{aligned}
DR_t^e &= \text{expected depreciation rate}, \\
\omega_t &= \text{exchange risk premium}.
\end{aligned} \tag{253}$$

Note that in the above derivation the uncovered interest rate parity relation includes a risk premium, ω_t:

$$R_t^{\mathrm{H}} = R_t^{\mathrm{F}} + RD_t^e + \omega_t. \tag{254}$$

The fundamental equation of the exchange rate thus helps us to identify the main causes of currency crises. Here are some examples:

Variable	Condition	Example
$\hat{b}_t^{\bar{\mathrm{H}}\mathrm{H}} - \hat{b}_t^{\bar{\mathrm{F}}\mathrm{F}}$	> 0	Hyperinflation, expansionary monetary policy
$\Delta y_t^{\mathrm{H}} - \Delta y_t^{\mathrm{F}}$	< 0	Low economic growth
ΔDR_t^e	> 0	Fear of devaluation
$\Delta \omega_t$	> 0	Risk of a suspension of payments
Δz_t^{HF}	< 0	Current account deficit
Δe_t^{HF}	> 0	Flight of foreign capital
Δb_t^{HF}	> 0	Payment of external debt

8.5.3 Economic Policy

Monetary Policy:

- Expansive monetary policy at home: low domestic interest rates, to avoid a recession for instance or to reduce unemployment
- Restrictive monetary policy abroad: high foreign interest rates, for example to reduce inflation

Fiscal policy:

- Budget deficit: high public expenditure and low taxes, for example to stimulate the economy, finance a war etc.
- $\Rightarrow$ Accumulation of public debt
- $\Rightarrow$ Monetization of public debt (for example, the central bank purchases governmental bonds)
- $\Rightarrow$ Increase in money supply (see above)

8.5.4 Balance of payments

Current account:

- Deficit financed immediately by money flows:

$$\Delta z_t^{\mathrm{HF}} < 0 \quad \Rightarrow \quad \Delta m_t^{\mathrm{HF}} < 0 \quad \Rightarrow \quad s_t \downarrow . \tag{255}$$

- Deficit financed by adaptive capital flows:

$$\Delta z_t^{\mathrm{HF}} < 0 \quad \Rightarrow \quad \Delta b_t^{\mathrm{HF}} < 0 \quad \Rightarrow \quad \Delta b_{t+1}^{\mathrm{HF}} > 0 \quad \Rightarrow \quad \Delta m_{t+1}^{\mathrm{HF}} < 0 \quad \Rightarrow \quad s_{t+1} \downarrow . \tag{256}$$

Current account deficits due to:

- Autonomous capital inflows and appreciation of the real exchange rate (case 7 above)
- Persistent inflation (case 8 above)
- Boom-and-bust cycles (case 9 above)

8.5.5 Measures to prevent currency crises

The fundamental equation of the exchange rate also points to certain measures to take in the event of a currency crisis:

Variable	Condition	Example of measure
$\hat{b}_t^{\bar{\mathrm{H}}\mathrm{H}} - \hat{b}_t^{\bar{\mathrm{F}}\mathrm{F}}$	< 0	Restrictive monetary policy, rising interest rates
$\Delta y_t^{\mathrm{H}} - \Delta y_t^{\mathrm{F}}$	> 0	Reforms to accelerate economic growth (difficult, slow)
ΔRD_t^e	< 0	Restoration of calm in the foreign exchange market
$\Delta \omega_t$	< 0	Transparency of public finances, loans from the IMF
Δz_t^{HF}	> 0	Restrictions on imports, moderate depreciation
Δe_t^{HF}	< 0	Capital controls, short-term restrictions on capital flows
Δb_t^{HF}	< 0	Renegotiation of foreign debt, loans from the IMF
$\Delta b_t^{\bar{\mathrm{H}}\bar{\mathrm{F}}}$	< 0	Sale of official reserves (also reduces the money supply)

9 Case studies

9.1 International debt crisis - 1980s

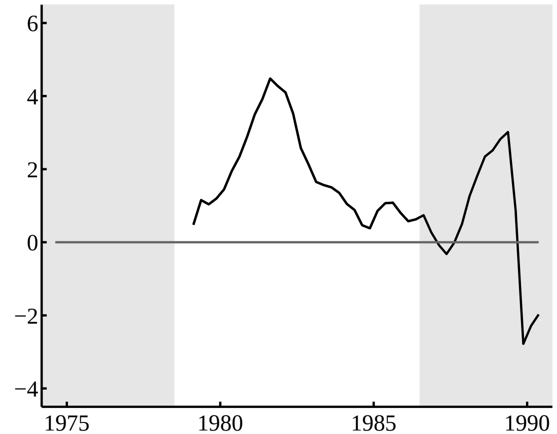
9.1.1 Mexico - 1982

- **Policies of economic and social development in the 1970s:** nationalization of the mining and electrical industries, redistribution of land and increased spending on health, housing construction, education and food subsidies
- Boosts in public spending facilitated by the **discovery in 1974 of vast oil fields** and the surge in the price of oil
- **Mexico borrowing heavily** from international capital markets
- **1977–1981: consumption rising by 7.6%** annually, **real investment by 17.2%, GDP by 8.6%**
- **Current account:**
  - **Largest deficit** in the world in **1981**
  - **Second-largest surplus** in the world in **1983**
- **Real exchange rate: rising until 1981,** then **falling by 50.3% between 1981 and 1987**

(a) Private consumption volume of Mexico (in logarithms)

(b) Investment volume of Mexico (in logarithms)

(c) GDP volume of Mexico (GDP 2005 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of Mexico (in billions of US dollars, annual moving average)

(e) Current account of Mexico (in billions of US dollars, annual moving average)

(f) Real effective exchange rate of Mexico (CPI-based, in logarithms)

Figure 1: **Case study: Mexico - 1982.**

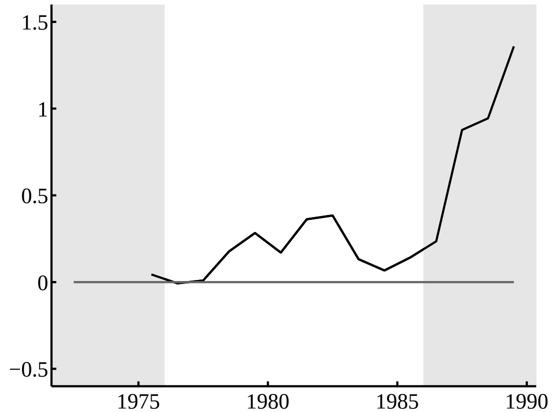
9.1.2 Chile - 1982

- Implementation of **market-oriented economic reforms** by the ”Chicago Boys” (since 1975): **privatization** of the **pension system** as well as **state-owned companies and banks, liberalization** of the country’s **current and financial accounts, consolidation of public finances** (while cutting taxes) and **stabilization of inflation.**
- **1976–1981: consumption rising by 9.2%** annually, **real investment by 15.2%, GDP by 7.5%**
- **Current account deficit of 14.5% of GDP in 1981**
- **Households and firms taking on great debts,** often in the form of foreign loans
- **Crash in 1982** following the hike in international interest rates:
  - **GDP dropping by 16.5% between 1981 and 1983**
  - **Largest per capita debt in Latin America**
- **Real exchange rate:**
  - **Rising by 30.0%** between 1980Q1 and 1982Q1
  - **Falling by 58.4%** between 1982Q1 and the end of the 1980s

(a) Private consumption volume of Chile (in logarithms)

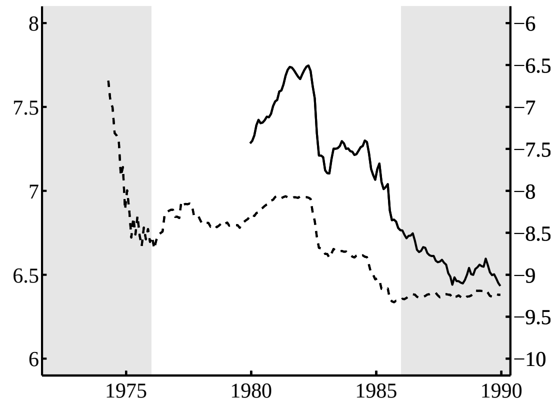
(b) Investment volume of Chile (in logarithms)

(c) GDP volume of Chile (GDP 2005 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of Chile (in billions of US dollars)

(e) Current account of Chile (in billions of US dollars)



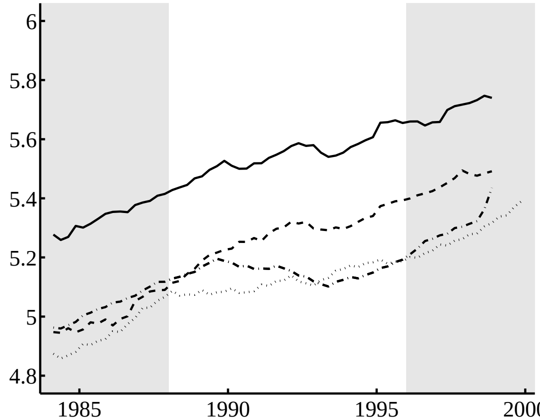
(f) Real effective exchange rate of Chile (CPI-based, in logarithms, solid line, left scale) and bilateral real exchange rate between Chile and the United States (CPI-based, in logarithms, dashed line, right scale)

Figure 2: **Case study: Chile - 1982.**

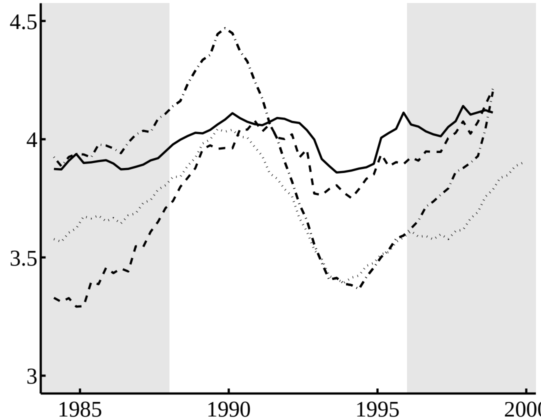
9.2 ERM crisis - 1992–1993

- September **1992: European Exchange Rate Mechanism (ERM)** enters a **severe crisis.**
- Crisis commonly explained by **self-fulfilling speculative attacks:**
  - **Speculation** against a weak currency forces **central bank** to **raise interest rates**
  - **Adverse effects** on **economic activity, employment, government's fiscal position, the banking system, firms' balance sheets, mortgage interest rates**
  - **Mere anticipation** of an exchange rate devaluation **forcing government to devalue indeed**
- **"Consensus interpretation"** among economic scholars (Eichengreen, 2003, chapter 8):

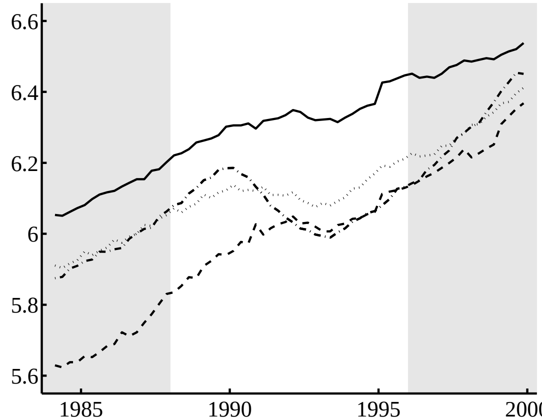
  "[T]here was **nothing inevitable** about the fact of the attacks, their timing, or their direction."
- However, empirical evidence shows that **ERM crisis** was **caused by external imbalances.**
- **Economic boom** in the affected countries from 1986 to 1989, **economic downturn** from 1989 to 1992
- Finland, France, Italy, Spain, Sweden and the United Kingdom were among the fourteen countries with the **highest current account deficits** in the 1990s, side by side with countries such as Mexico, Brazil and Korea



(a) Private consumption volume (in logarithms) of Italy (solid line), Spain (dashed line), Sweden (dotted line) and Finland (dash-dotted line)

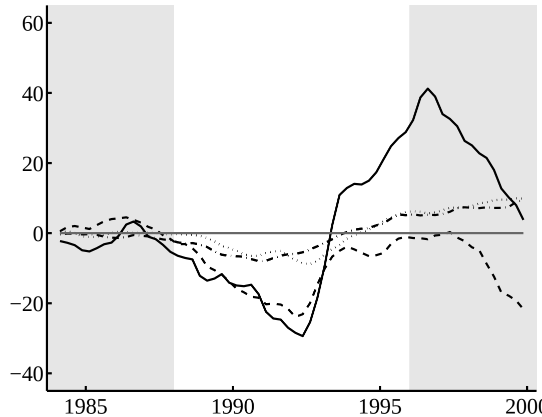


(b) Investment volume (in logarithms) of Italy (solid line), Spain (dashed line), Sweden (dotted line) and Finland (dash-dotted line)

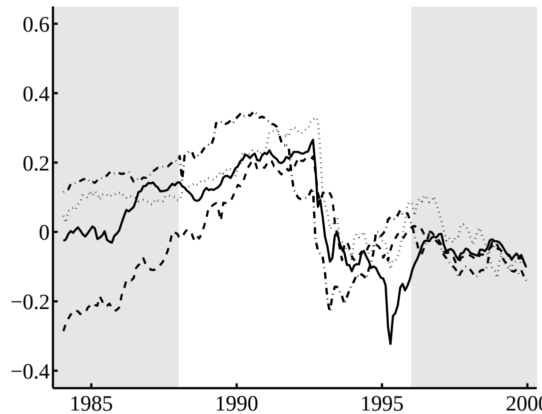


(c) GDP volume (GDP 2005 = 100, in logarithms) of Italy (solid line), Spain (dashed line), Sweden (dotted line) and Finland (dash-dotted line)

(d) Net direct and portfolio investment inflows (in billions of US dollars, annual moving average) of Italy (solid line), Spain (dashed line), Sweden (dotted line) and Finland (dash-dotted line)



(e) Current accounts (in billions of US dollars, annual moving average) of Italy (solid line), Spain (dashed line), Sweden (dotted line) and Finland (dash-dotted line)



(f) Real effective exchange rate (CPI-based, in logarithms) of Italy (solid line), Spain (dashed line), Sweden (dotted line) and Finland (dash-dotted line)

Figure 3: **Case study: ERM crisis - 1992 - Italy, Spain, Sweden and Finland.**

9.2.1 Germany and the Netherlands

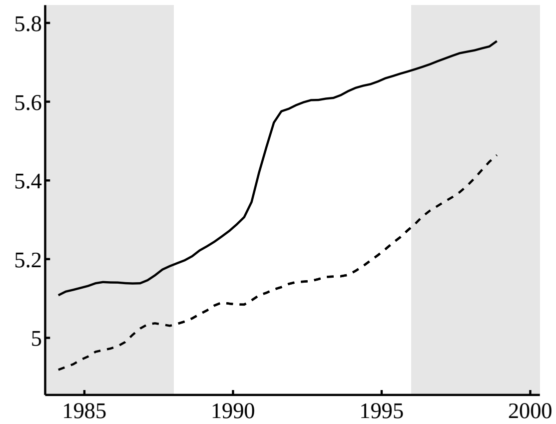
Why did **Germany and the Netherlands not have to devalue?**

Common explanation:

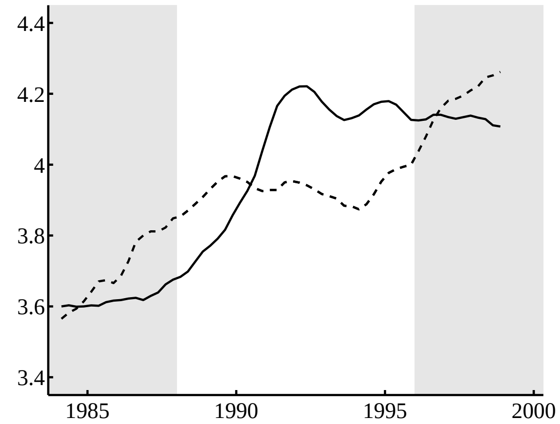
- **Dutch central bank's** determination to **follow Germany's monetary policy** slavishly (Obstfeld and Rogoff, 1995)

Alternative explanation put forward here:

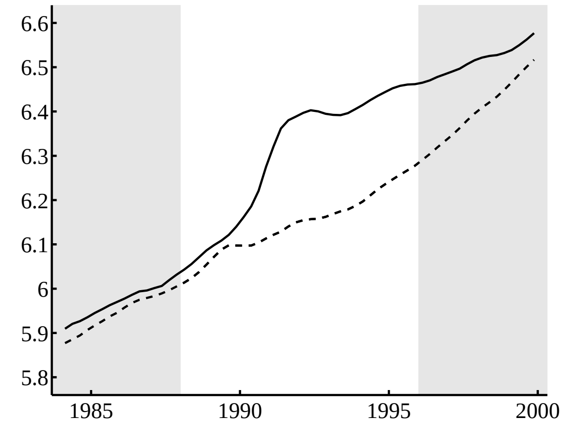
- Both countries running **very large current account surpluses** in the years prior to the crisis
  - **Germany's** current account **surplus** was the **highest in the world** in 1990 (of 146 countries)
  - **Dutch** current account **surplus** ranked **third** in 1989 (of 145 countries) and second in the mid-1990s (of 158–159 countries, after that of Japan).



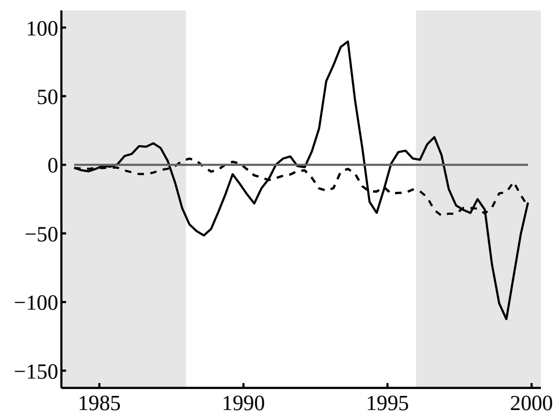
(a) Private consumption volume (in logarithms) of Germany (solid line) and the Netherlandds (dashed line)



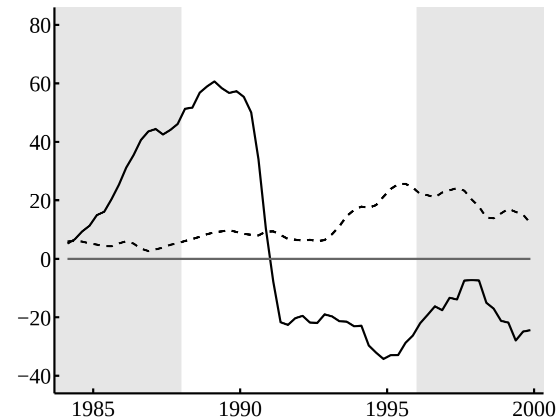
(b) Investment volume (in logarithms) of Germany (solid line) and the Netherlandds (dashed line)



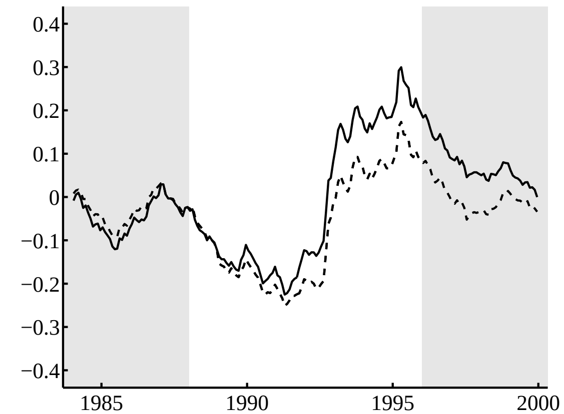
(c) GDP volume (GDP 2005 = 100, in logarithms) of Germany (solid line) and the Netherlandds (dashed line)



(d) Net direct and portfolio investment inflows (in billions of US dollars, annual moving average) of Germany (solid line) and the Netherlands (dashed line)

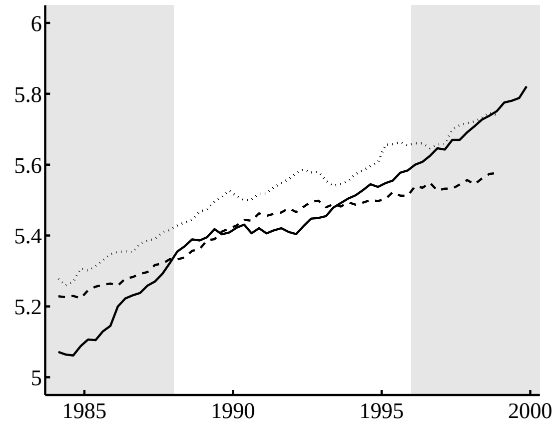


(e) Current accounts (in billions of US dollars, annual moving average) of Germany (solid line) and the Netherlands (dashed line)

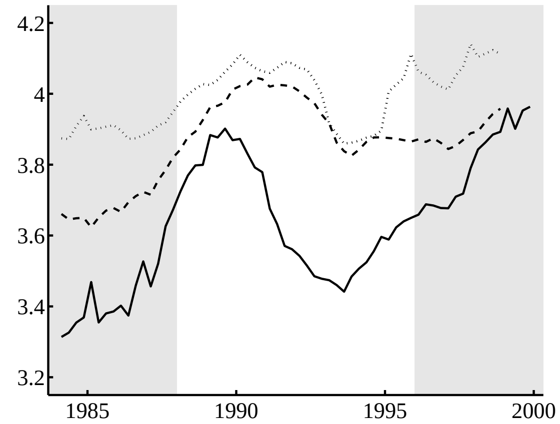


(f) Real effective exchange rate of Germany (solid line) and the Netherlands (dashed line) versus the geometric average of the United Kingdom, France, Italy, Spain, Sweden and Finland (CPI-based, in logarithms)

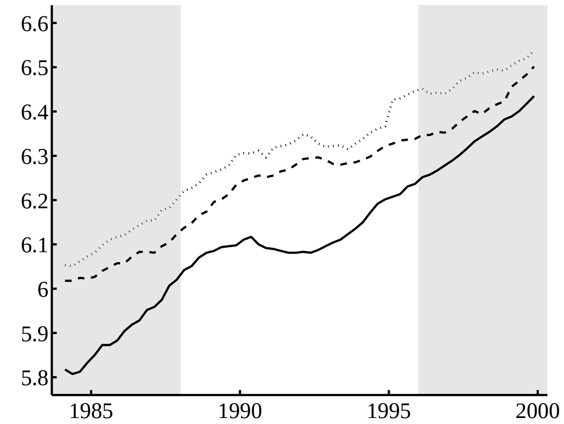
Figure 4: **Case study: ERM crisis - 1992 - Germany and the Netherlands.** Unlike other European currencies, the German mark and the Dutch guilder revalued during the ERM crisis. Previously, Germany and the Netherlands had been running very large current account surpluses for several years.



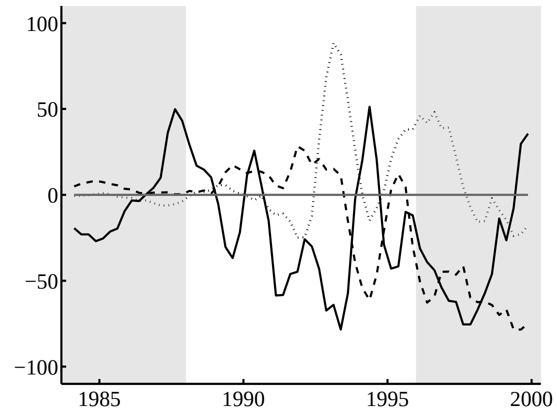
(a) Private consumption volume (in logarithms) of the United Kingdom (solid line), France (dashed line) and Italy (dotted line)



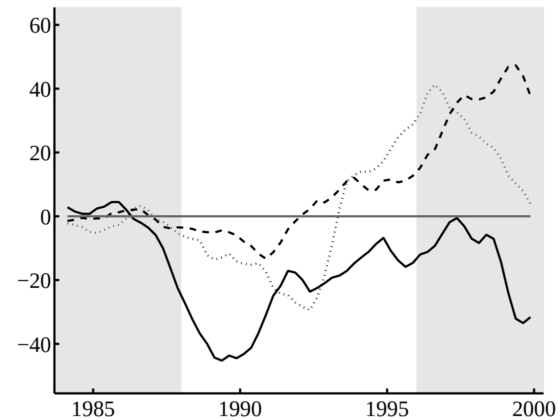
(b) Investment volume (in logarithms) of the United Kingdom (solid line), France (dashed line) and Italy (dotted line)



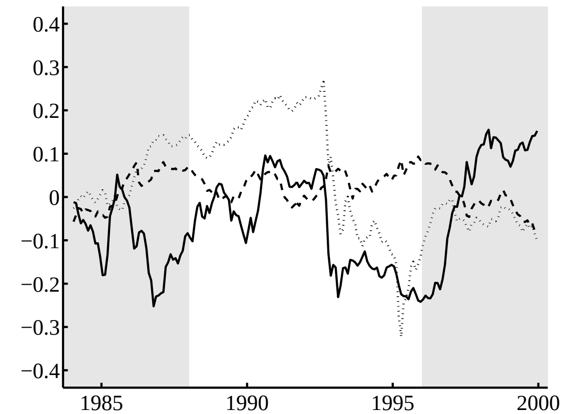
(c) GDP volume (GDP 2005 = 100, in logarithms) of the United Kingdom (solid line), France (dashed line) and Italy (dotted line)



(d) Net direct and portfolio investment inflows (in billions of US dollars, annual moving average) of the United Kingdom (solid line), France (dashed line) and Italy (dotted line)

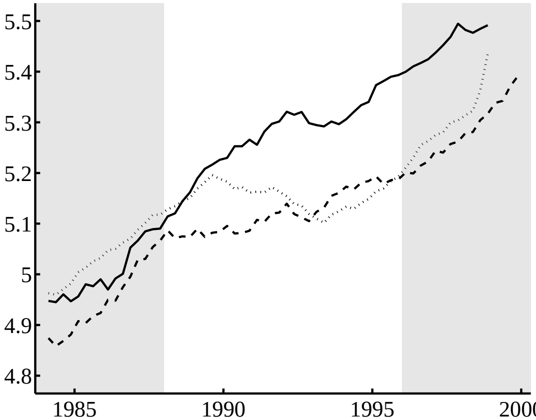


(e) Current accounts (in billions of US dollars, annual moving average) of the United Kingdom (solid line), France (dashed line) and Italy (dotted line)

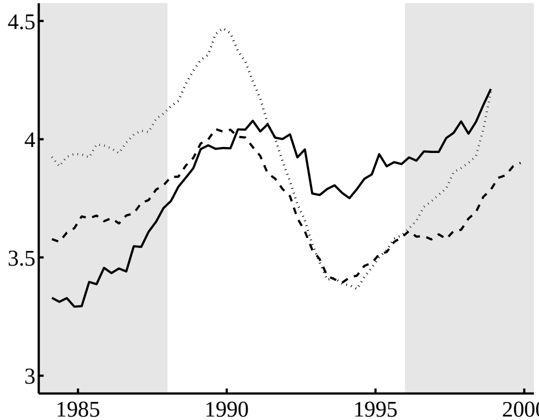


(f) Real effective exchange rate (CPI-based, in logarithms) of the United Kingdom (solid line), France (dashed line) and Italy (dotted line)

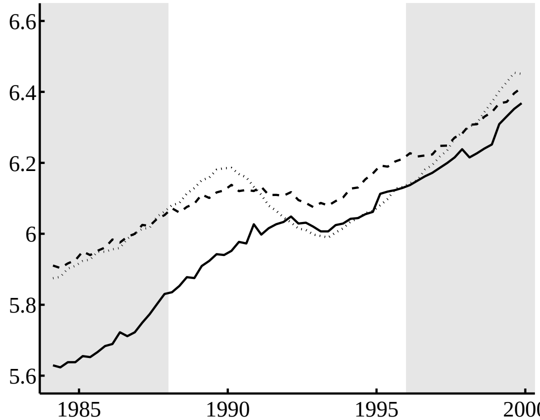
Figure 5: **Case study: ERM crisis - 1992 - United Kingdom, France and Italy.**



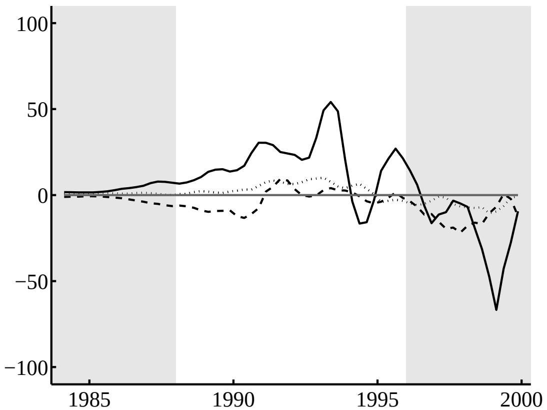
(a) Private consumption volume (in logarithms) of Spain (solid line), Sweden (dashed line) and Finland (dotted line)



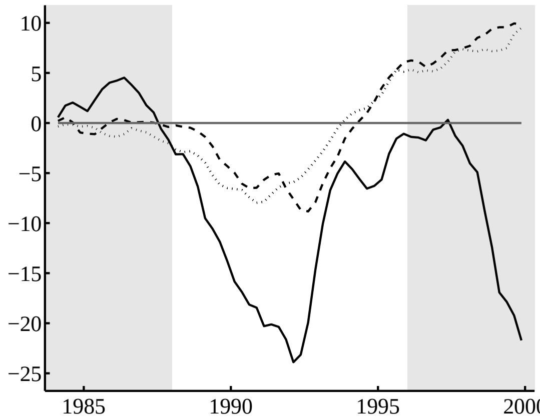
(b) Investment volume (in logarithms) of Spain (solid line), Sweden (dashed line) and Finland (dotted line)



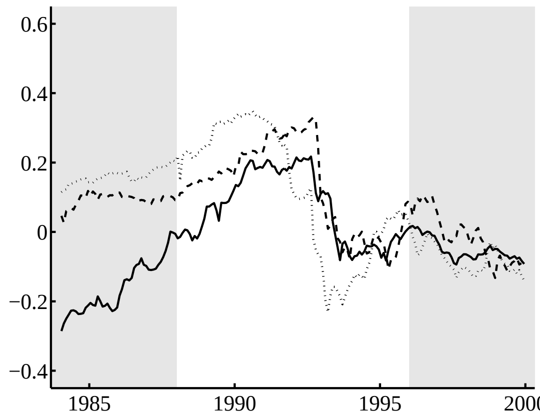
(c) GDP volume (GDP 2005 = 100, in logarithms) of Spain (solid line), Sweden (dashed line) and Finland (dotted line)



(d) Net direct and portfolio investment inflows (in billions of US dollars, annual moving average) of Spain (solid line), Sweden (dashed line) and Finland (dotted line)

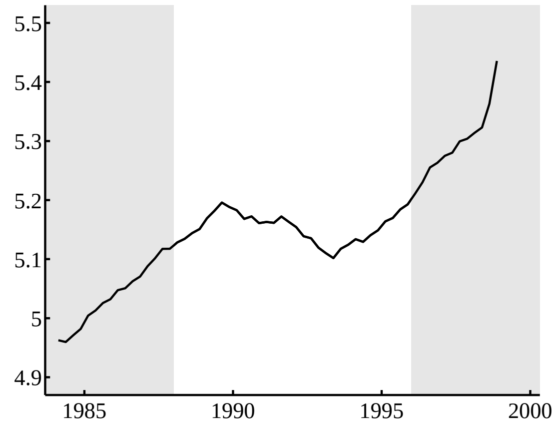


(e) Current accounts (in billions of US dollars, annual moving average) of Spain (solid line), Sweden (dashed line) and Finland (dotted line)

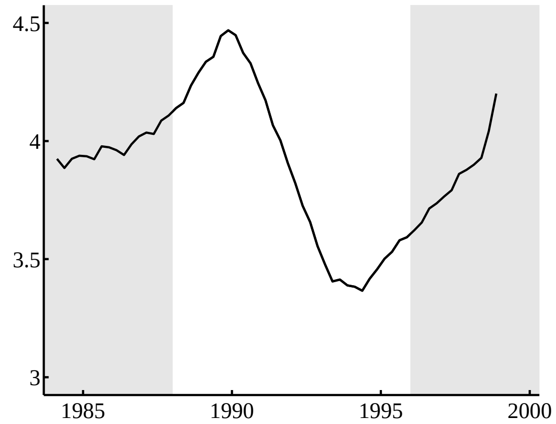


(f) Real effective exchange rate (CPI-based, in logarithms) of Spain (solid line), Sweden (dashed line) and Finland (dotted line)

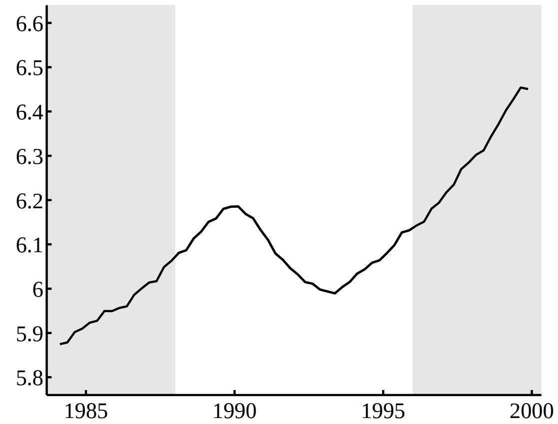
Figure 6: **Case study: ERM crisis - 1992 - Spain, Sweden and Finland.**



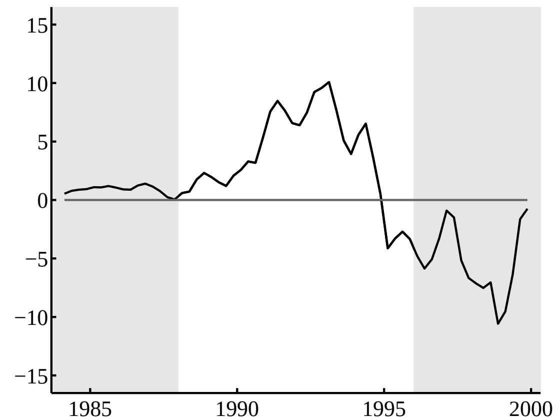
(a) Private consumption volume of Finland (in logarithms, annual moving average)



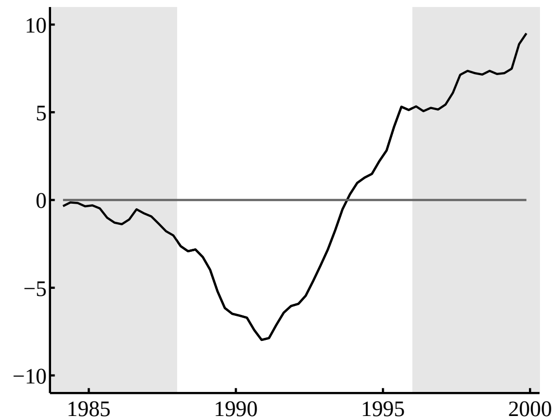
(b) Investment volume of Finland (in logarithms, annual moving average)



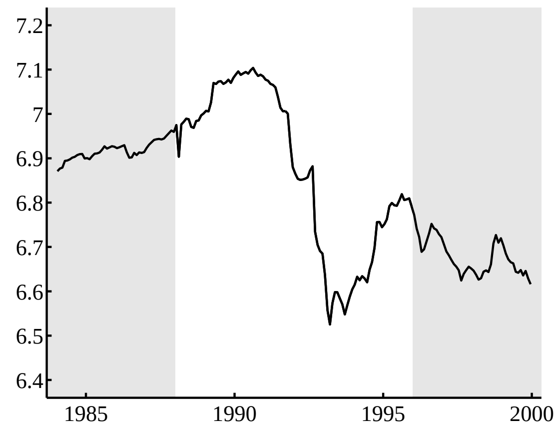
(c) GDP volume of Finland (GDP 2005 = 100, in logarithms, annual moving average)



(d) Net direct and portfolio investment inflows of Finland (in billions of US dollars, annual moving average)

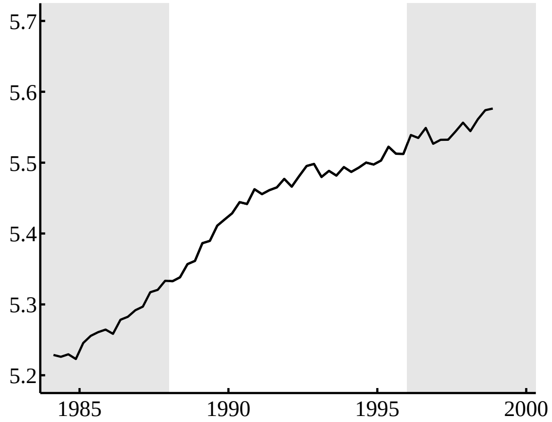


(e) Current account of Finland (in billions of US dollars, annual moving average)

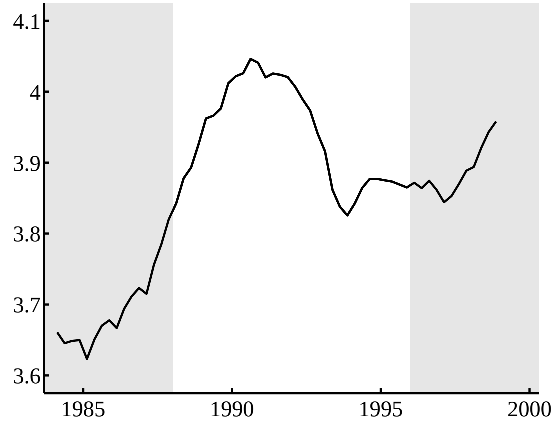


(f) Real effective exchange rate of Finland (CPI-based, in logarithms)

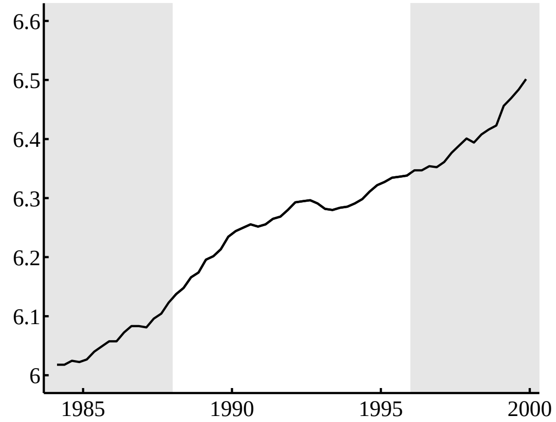
Figure 7: **Case study: Finland - 1992.**



(a) Private consumption volume of France (seasonally adjusted, in logarithms)

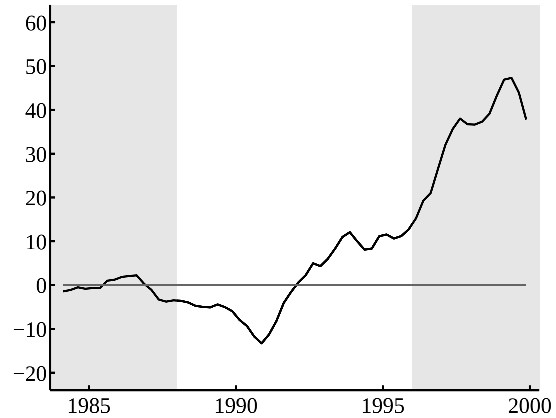


(b) Investment volume of France (seasonally adjusted, in logarithms)

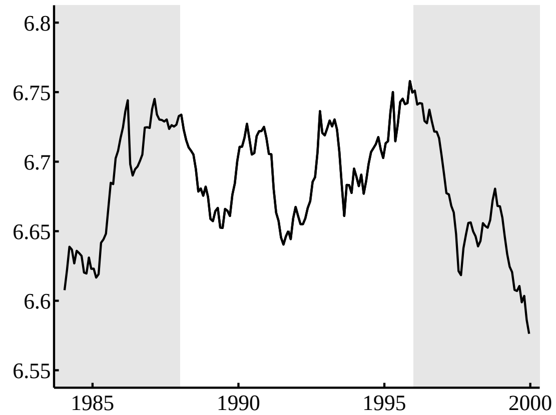


(c) GDP volume of France (GDP 2005 = 100, seasonally adjusted, in logarithms)

(d) Net direct and portfolio investment inflows of France (in billions of US dollars, annual moving average)

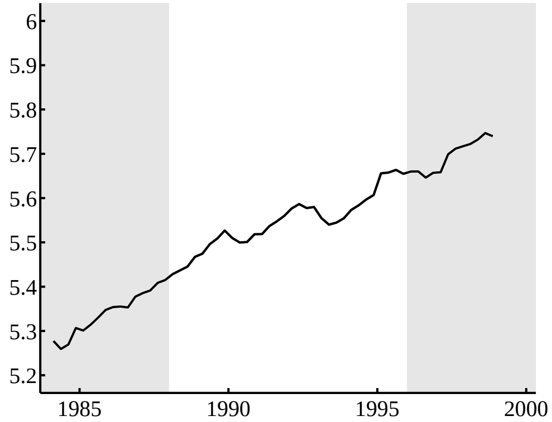


(e) Current account of France (in billions of US dollars)



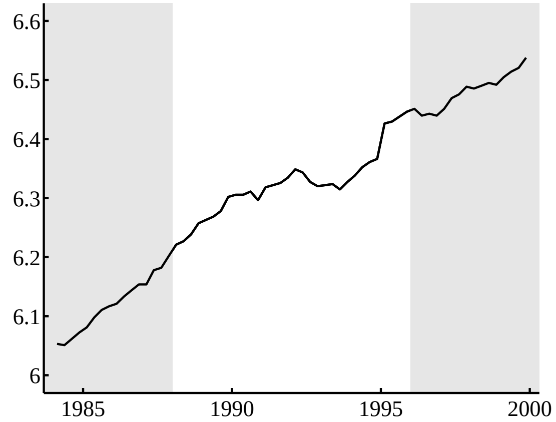
(f) Real effective exchange rate of France (CPI-based, in logarithms)

Figure 8: **Case study: France - 1992.**



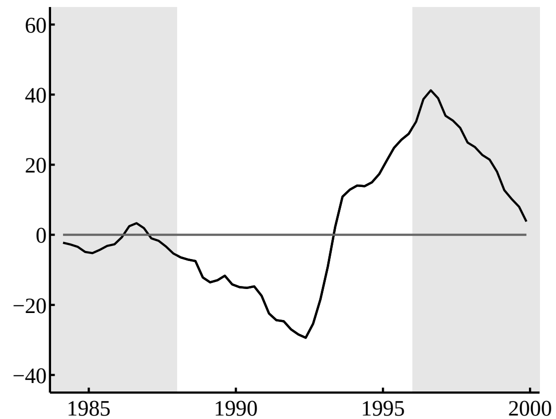
(a) Private consumption volume of Italy (in logarithms)

(b) Investment volume of Italy (in logarithms)



(c) GDP volume of Italy (GDP 2005 = 100, in logarithms)

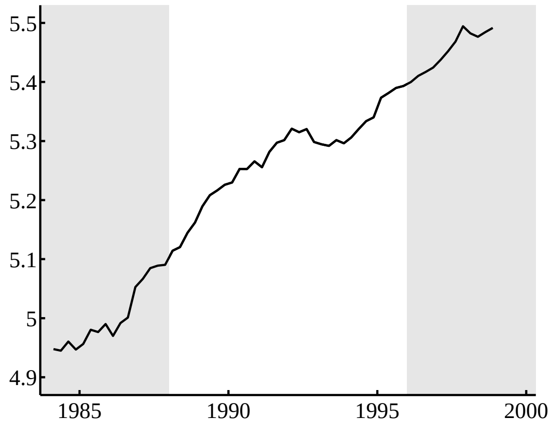
(d) Net direct and portfolio investment inflows of Italy (in billions of US dollars, annual moving average)



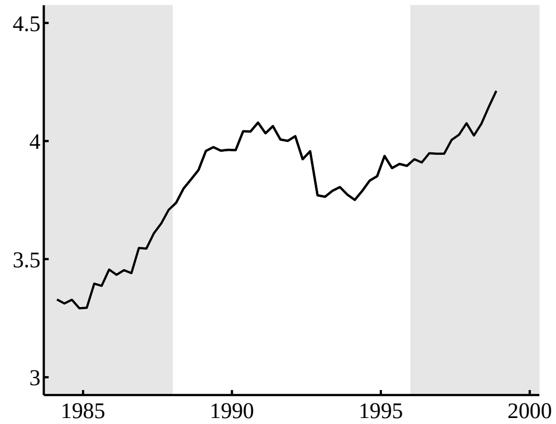
(e) Current account of Italy (in billions of US dollars, annual moving average)

(f) Real effective exchange rate of Italy (CPI-based, in logarithms)

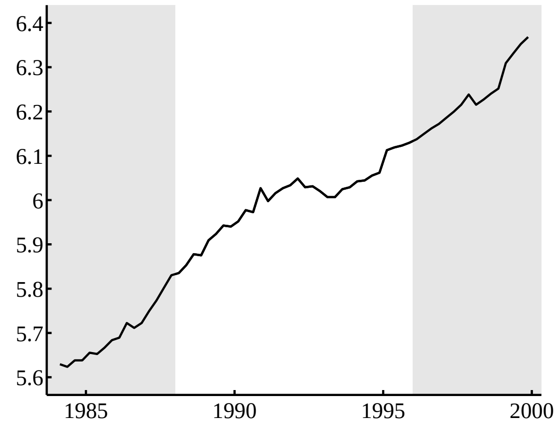
Figure 9: **Case study: Italy - 1992.**



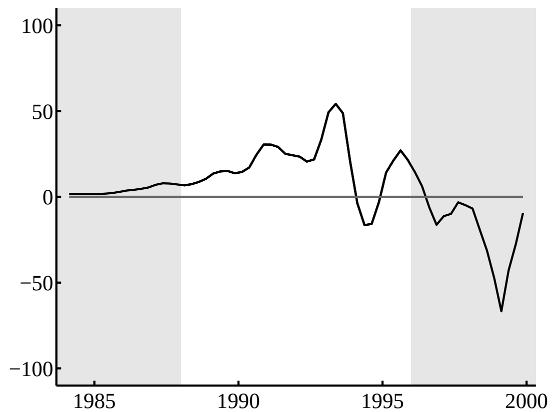
(a) Private consumption volume of Spain (seasonally adjusted, in logarithms)



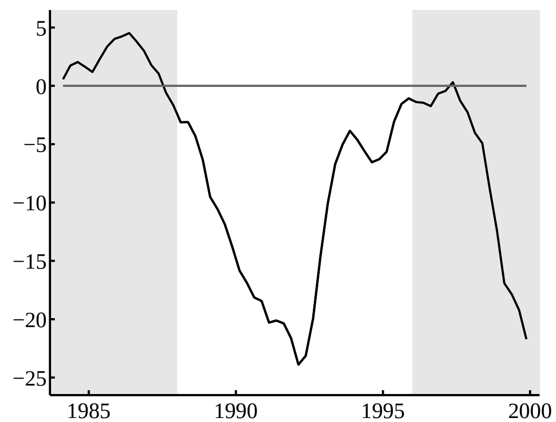
(b) Investment volume of Spain (seasonally adjusted, in logarithms)



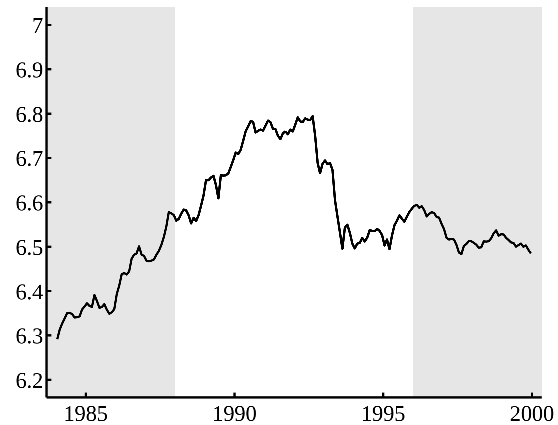
(c) GDP volume of Spain (GDP 2005 = 100, seasonally adjusted, in logarithms)



(d) Net direct and portfolio investment inflows of Spain (in billions of US dollars, annual moving average)

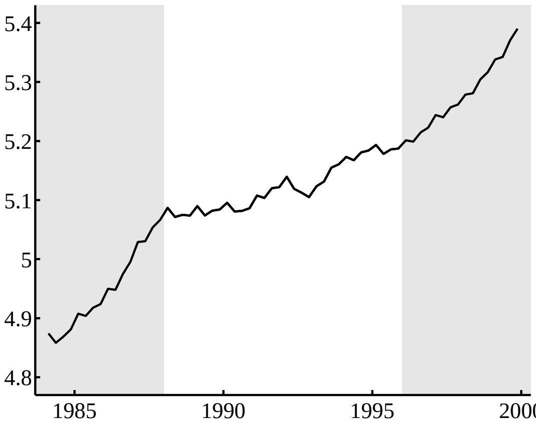


(e) Current account of Spain (in billions of US dollars, annual moving average)

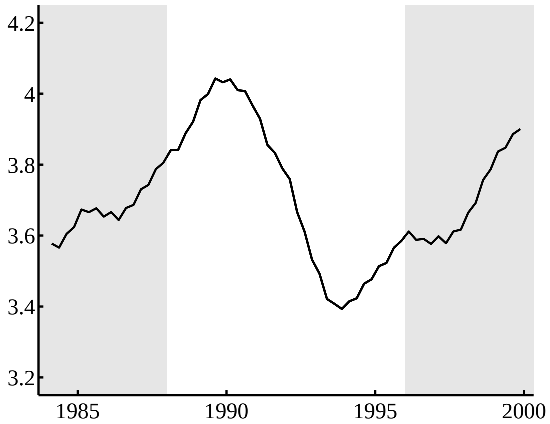


(f) Real effective exchange rate of Spain (CPI-based, in logarithms)

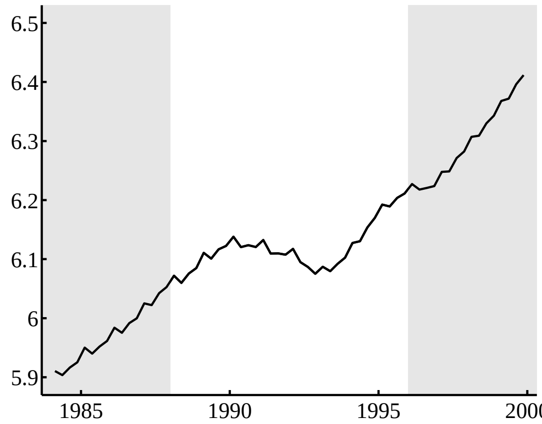
Figure 10: **Case study: Spain - 1992.**



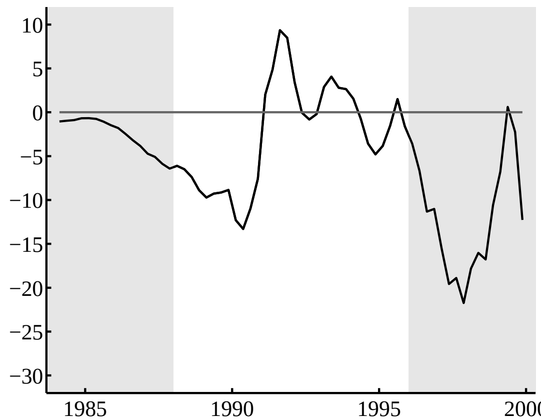
(a) Private consumption volume of Sweden (in logarithms)



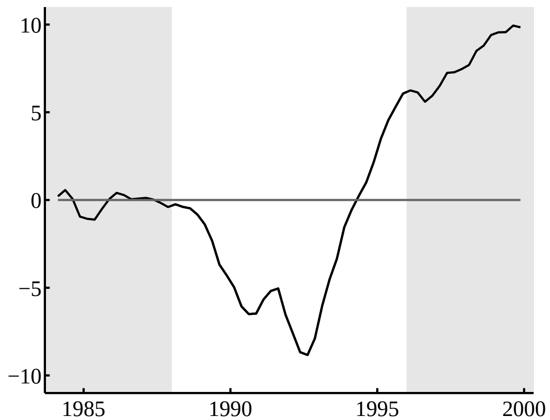
(b) Investment volume of Sweden (in logarithms)



(c) GDP volume of Sweden (GDP 2005 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of Sweden (in billions of US dollars, annual moving average)



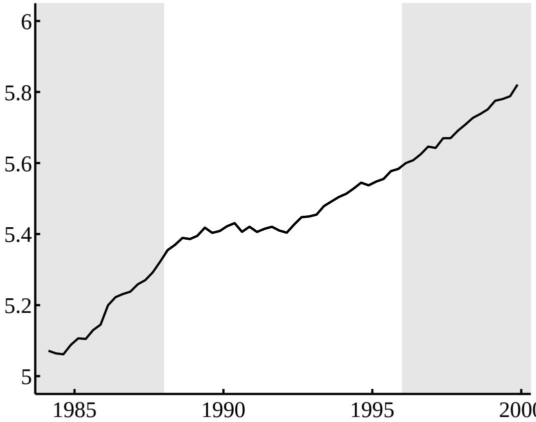
(e) Current account of Sweden (in billions of US dollars, annual moving average)

(f) Real effective exchange rate of Sweden (CPI-based, in logarithms)

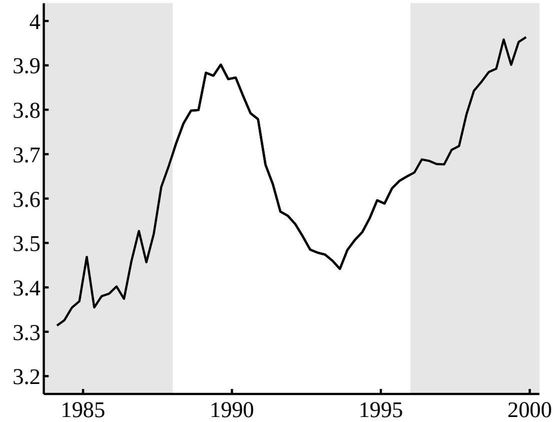
Figure 11: **Case study: Sweden - 1992.**

9.2.2 United Kingdom - 1992

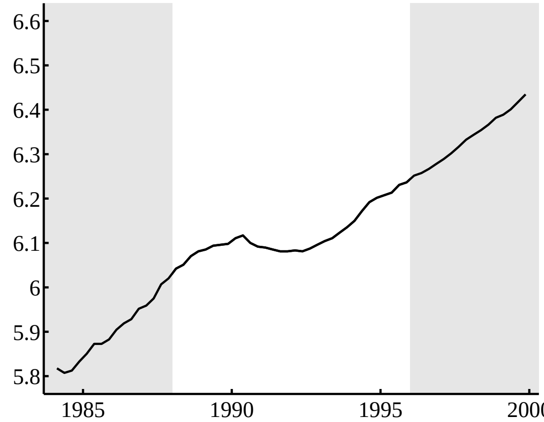
- Margaret Thatcher pursuing **economic reforms in the 1980s** (influenced by the monetarist and supply-side economics ideologies of the time): **rise in interest rates, fiscal consolidation, tax reductions for high-income earners, liberalization of the financial sector, privatization of state-owned industries, crushing of the trade unions**
- **”Lawson boom”** until the recession of 1992: **strong growth** in the second half of the 1980s (around 4% in real terms), accompanied by **falling unemployment** (from from 11.8% in 1984 to 7.0% in 1990) and a **boom in residential and commercial real estate**
- **Second-largest current account deficit in the world** from 1988 to 1990 due to surge in consumption and investment
- **Exchange rate:**
  - United Kingdom **joining the European Exchange Rate Mechanism (ERM) in October 1990** and **abandoning it on 16 September 1992** (”Black Wednesday”)
  - **Real exchange rate appreciating since mid-1986,** yet **very low for several years after the devaluation in late 1992.**



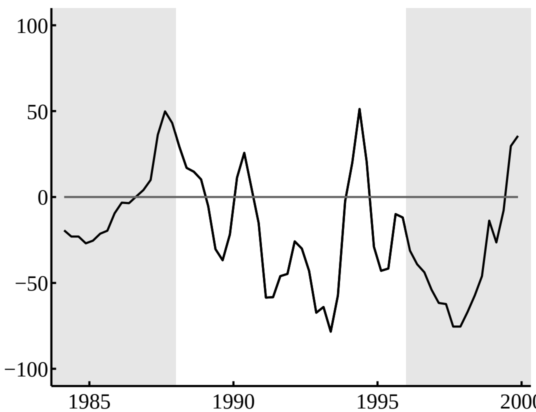
(a) Private consumption volume of the United Kingdom (in logarithms)



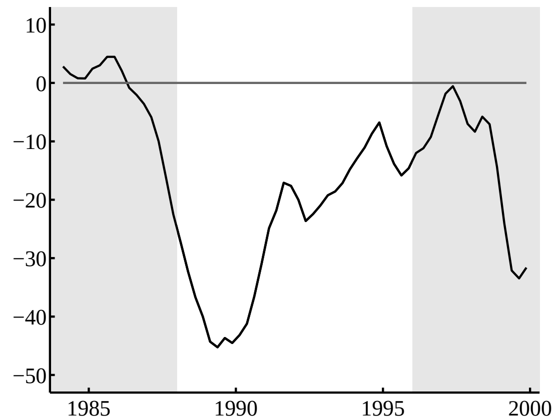
(b) Investment volume of the United Kingdom (in logarithms)



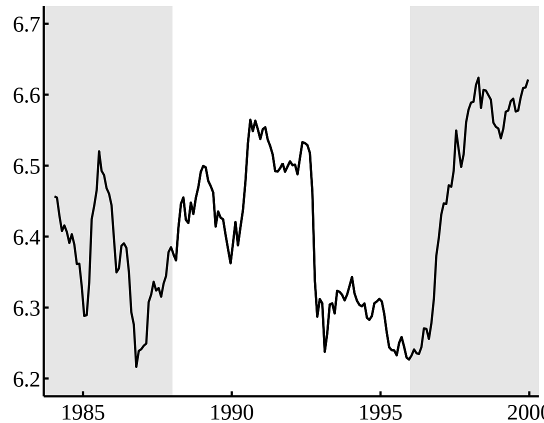
(c) GDP volume of the United Kingdom (GDP 2005 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of the United Kingdom (in billions of US dollars, annual moving average)



(e) Current account of the United Kingdom (in billions of US dollars, annual moving average)

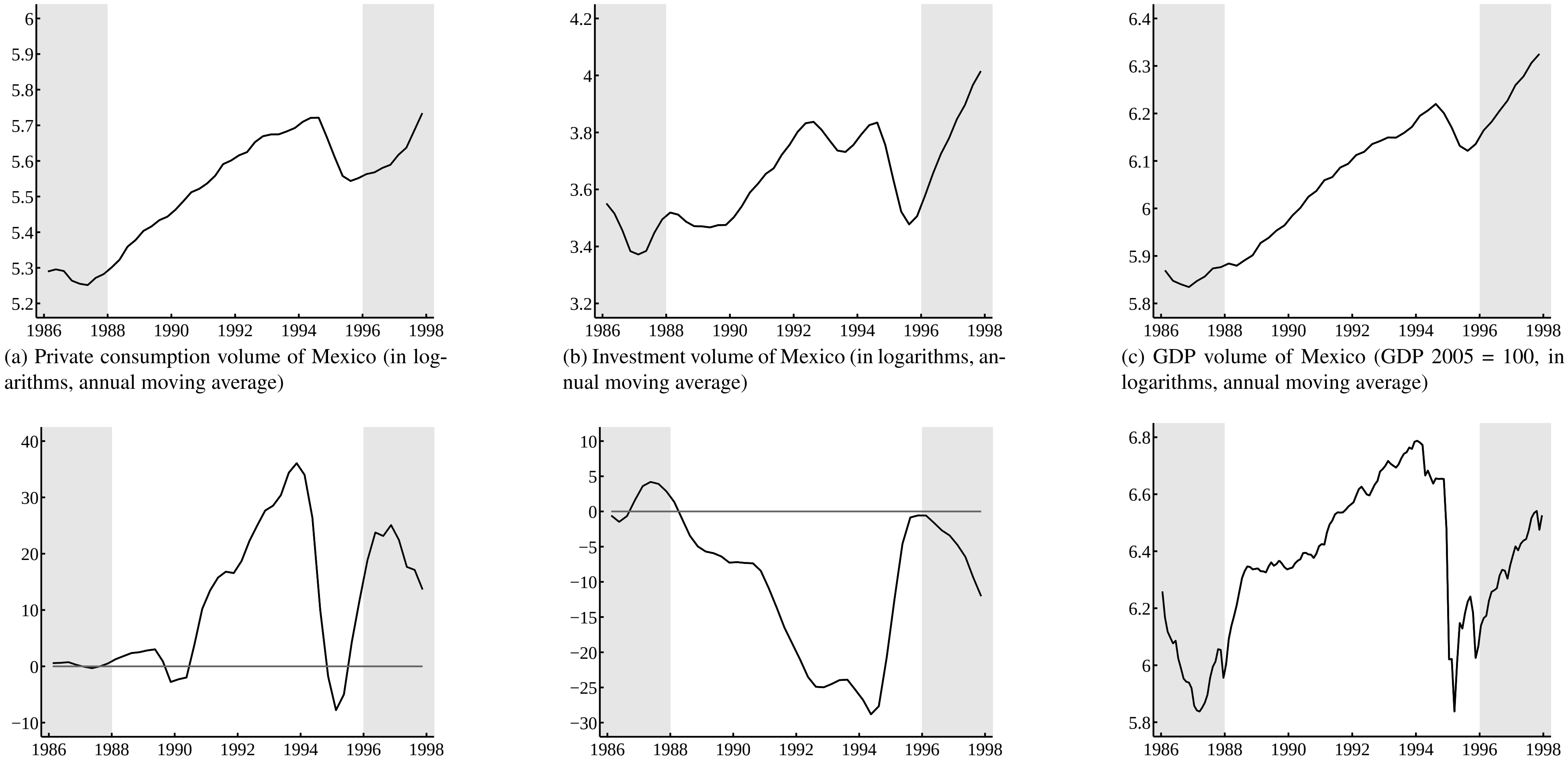


(f) Real effective exchange rate of the United Kingdom (CPI-based, in logarithms)

Figure 12: **Case study: United Kingdom - 1992.**

9.3 Mexico - 1994

- **Mexico's "tequila crisis" of 1994–1995 after** a period of good macroeconomic performance, which followed the **implementation of a stabilization programme, privatization policies and structural reforms in the mid-1980s.**
- **1988–1994: consumption rising by 4.9%** per year in real terms, **investment by 4.7%** and **GDP by 3.9%**
- **Current account deficit** reaching 6.8% of GDP in 1992 - **second-largest deficit in the world** in 1993–1994
- Massive **capital inflows** in the first half of the 1990s
- **Large debts of the private and public sectors**
- **Exchange rate:**
  - **Real exchange rate almost doubling** in the years leading up to the crisis
  - **Crawling peg** with the dollar **abandoned in December 1994,** initiating a **50% nominal depreciation over the next six months**



(a) Private consumption volume of Mexico (in logarithms, annual moving average)

(b) Investment volume of Mexico (in logarithms, annual moving average)

(c) GDP volume of Mexico (GDP 2005 = 100, in logarithms, annual moving average)

(d) Net direct and portfolio investment inflows of Mexico (in billions of US dollars, annual moving average)

(e) Current account of Mexico (in billions of US dollars, annual moving average)

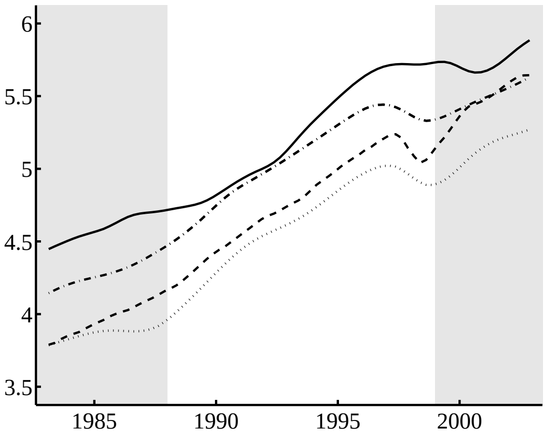
(f) Real effective exchange rate of Mexico (CPI-based, in logarithms)

Figure 13: **Case study: Mexico - 1994.**

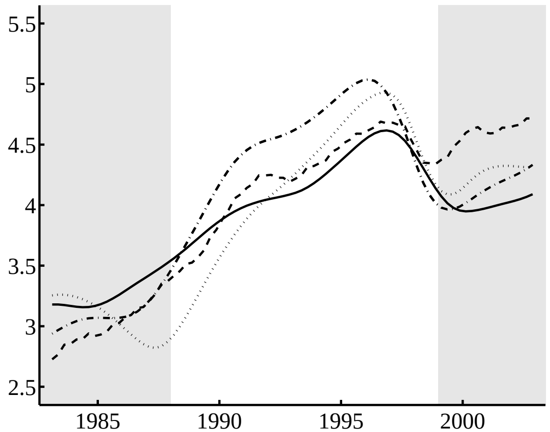
9.4 Asian crisis - 1997–1998

	Indonesia	Korea	Malaysia	Thailand
Boom: 1988-1996				
Real consumption (per year)	8.4%	8.5%	8.5%	8.3%
Real investment (per year)	9.0%	10.9%	17.2%	13.1%
Real GDP (per year)	7.3%	7.8%	9.4%	9.0%
Bust: 1997–1998				
Real consumption (per year)	-4.5%	-11.0%	-15.0%	-11.3%
Real investment (per year)	-21.9%	-23.3%	-42.3%	-40.7%
Real GDP (per year)	-13.1%	-6.9%	-7.3%	-10.5%

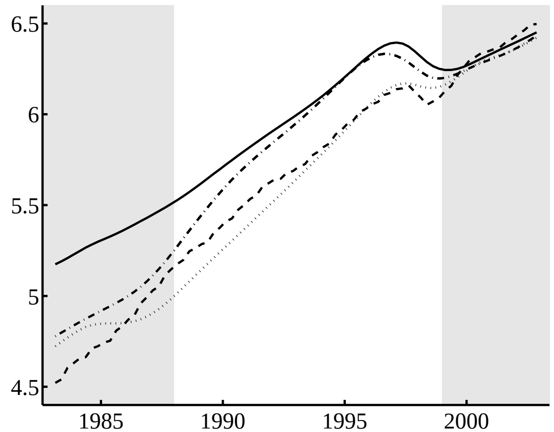
- Large **current account deficits:** Korea running the third-largest current account deficit in the world in 1996 (of 160 countries) and the second-largest *surplus* in 1998 (of 162 countries)
- **Capital inflows:** capital account **liberalization, high returns, stock market boom,** rise in portfolios of **institutional investors** and **mutual funds** and internationalization of **foreign banks**
- **Crisis:** due to **large and persistent current account deficits, not (!)** to **capital outflows**



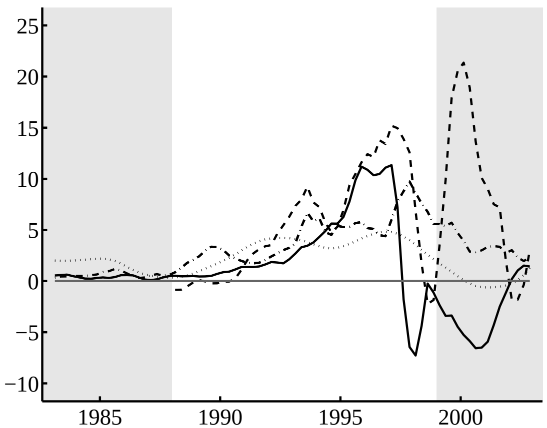
(a) Private consumption volume (in logarithms) of Indonesia (solid line), Korea (dashed line), Malaysia (dotted line) and Thailand (dash-dotted line)



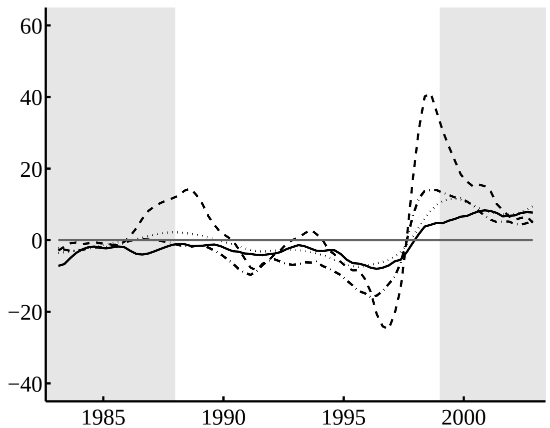
(b) Investment volume (in logarithms) of Indonesia (solid line), Korea (dashed line), Malaysia (dotted line) and Thailand (dash-dotted line)



(c) GDP volume (GDP 2005 = 100, in logarithms) of Indonesia (solid line), Korea (dashed line), Malaysia (dotted line) and Thailand (dash-dotted line)



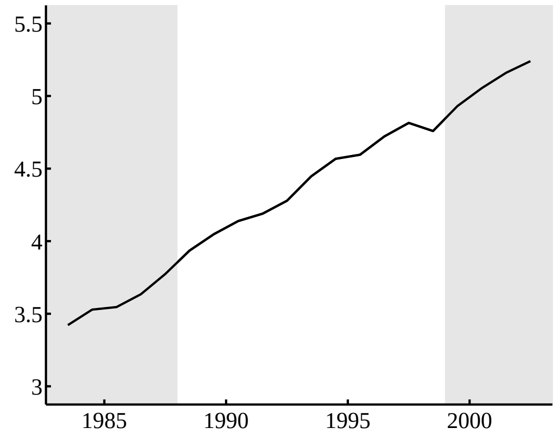
(d) Net direct and portfolio investment inflows (in billions of US dollars, annual moving average) of Indonesia (solid line), Korea (dashed line), Malaysia (dotted line) and Thailand (dash-dotted line)



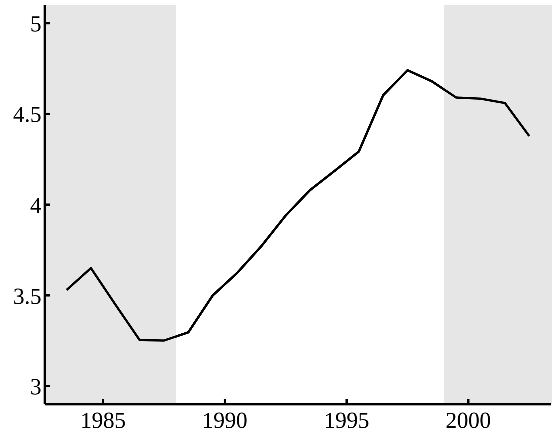
(e) Current account (in billions of US dollars, annual moving average) of Indonesia (solid line), Korea (dashed line), Malaysia (dotted line) and Thailand (dash-dotted line)

(f) Real effective exchange rate (in logarithms) of Indonesia (solid line), Korea (dashed line), Malaysia (dotted line) and Thailand (dash-dotted line)

Figure 14: **Case study: Asian crisis - 1997.**

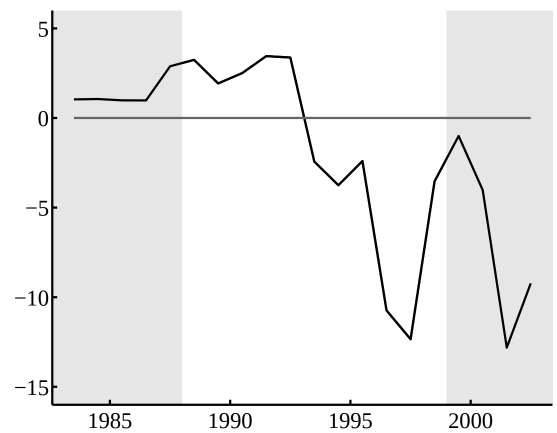


(a) Private consumption volume of Singapore (in logarithms)

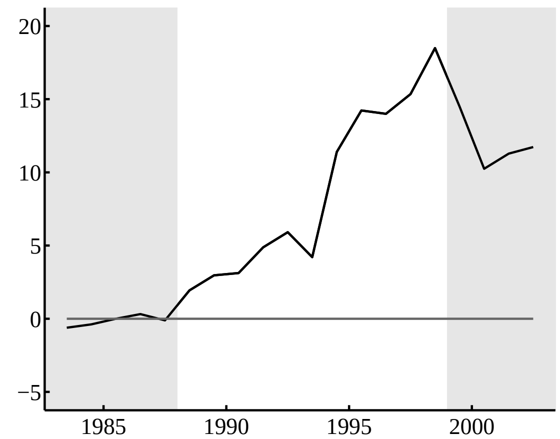


(b) Investment volume of Singapore (in logarithms)

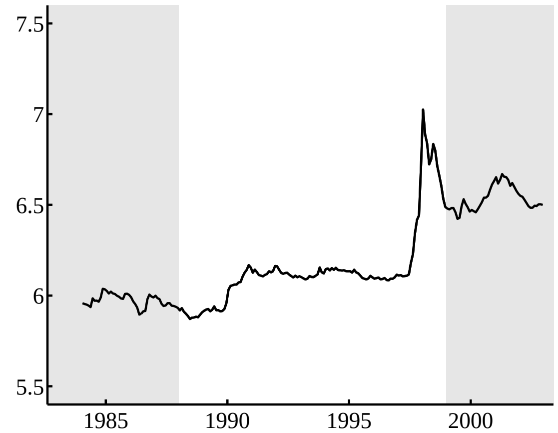
(c) GDP volume of Singapore (GDP 2005 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of Singapore (in billions of US dollars)



(e) Current account of Singapore (in billions of US dollars)

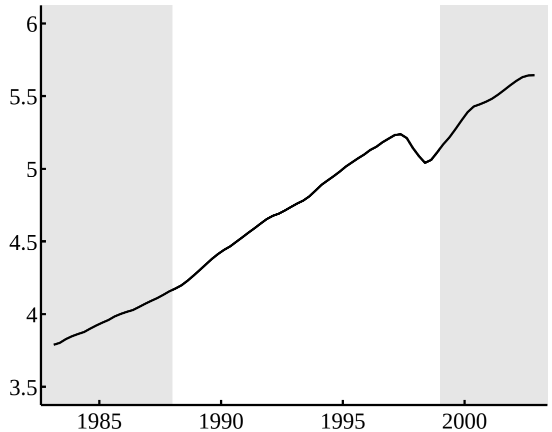


(f) Real effective exchange rate of Singapore versus the geometric average of Indonesia, Korea, Malaysia, the Philippines and Thailand (in logarithms)

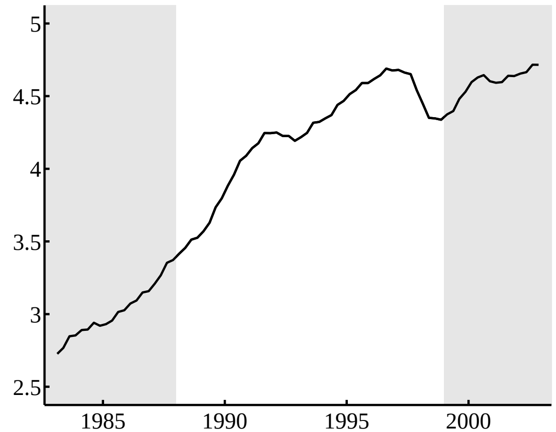
Figure 15: **Case study: Singapore - 1997.** Unlike the currencies of neighbouring countries, the Singapore dollar revalued during the Asian crisis. Previously, Singapore had been running large current account surpluses for several years.

9.4.1 Korea - 1997

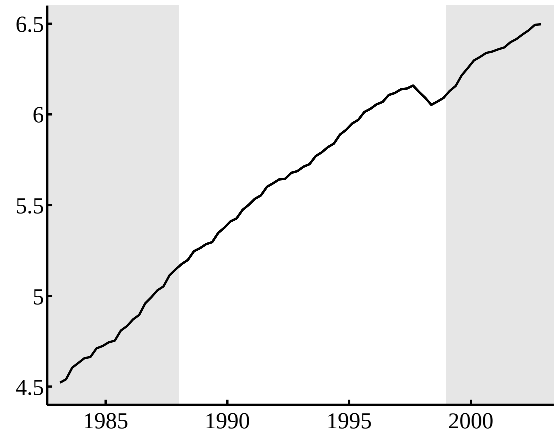
- From around 1991: **capital inflows** on an unprecedented scale
  - **Liberalization** of the country's **financial account**
  - Growing importance of **institutional investors and mutual funds**
  - **Low interest rates in the developed world** in the early 1990s
- **1988–1996: private consumption growing by 8.5%** per year in real terms, **investment by firms by 10.9%** and **overall production by 7.8%**
- **Current account:**
  - **Third-largest deficit** in the world in 1996
  - **Second-largest surplus** in the world in 1998 (equivalent to 10.2% of GDP).
- **Exchange rate:**
  - **Appreciation** of the Korean won **by 41.7% in real terms** between 1986 and 1996
  - Dollar exchange rate of the won dropping by half in 1997, contributing to a **trade-weighted real depreciation of 39.5%** of the Korean currency between 1996 and 1998



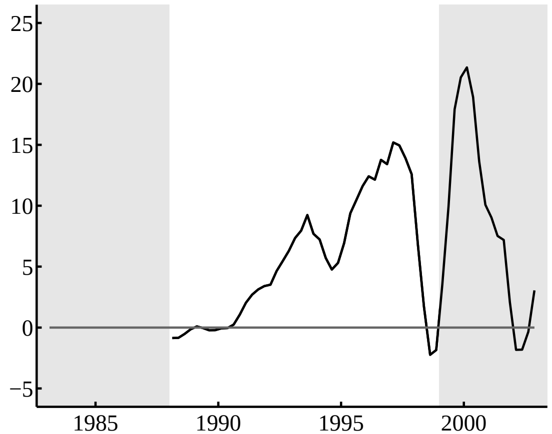
(a) Private consumption volume of Korea (in logarithms, annual moving average)



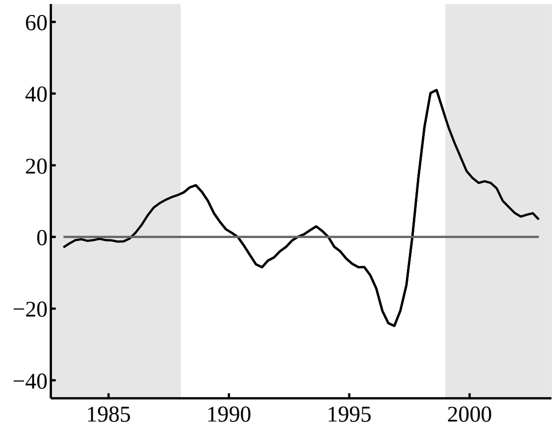
(b) Investment volume of Korea (in logarithms, annual moving average)



(c) GDP volume of Korea (GDP 2005 = 100, in logarithms, annual moving average)



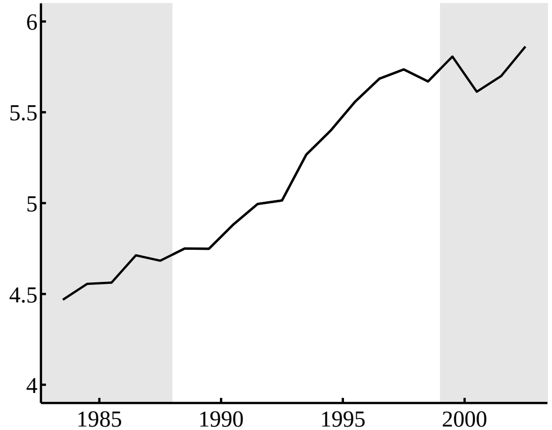
(d) Net direct and portfolio investment inflows of Korea (in billions of US dollars, annual moving average)



(e) Current account of Korea (in billions of US dollars, annual moving average)

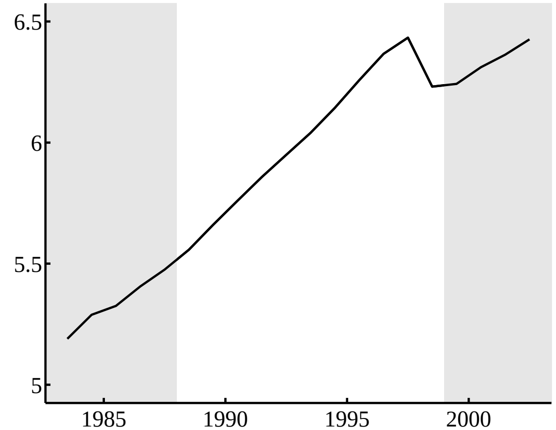
(f) Real effective exchange rate of Korea (ULC-based, in logarithms)

Figure 16: **Case study: Korea - 1997.**

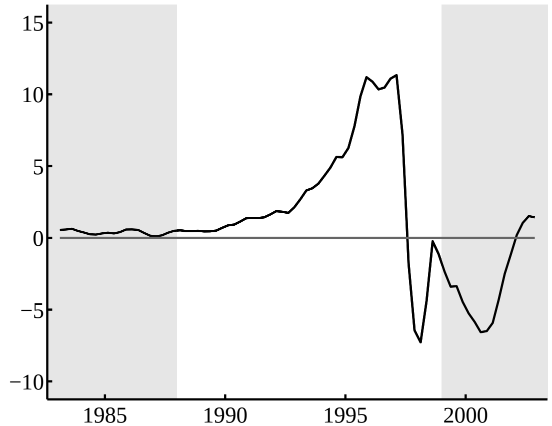


(a) Private consumption volume of Indonesia (in logarithms)

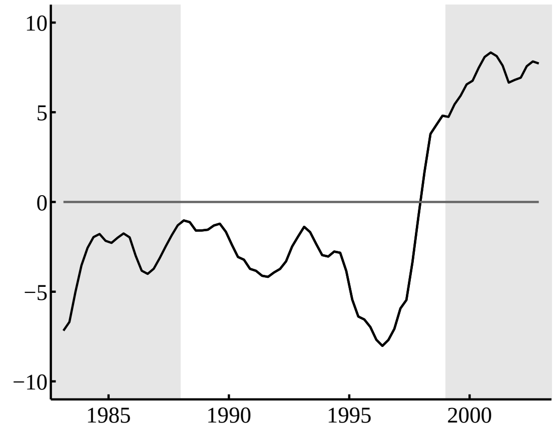
(b) Investment volume of Indonesia (in logarithms)



(c) GDP volume of Indonesia (GDP 2005 = 100, in logarithms)



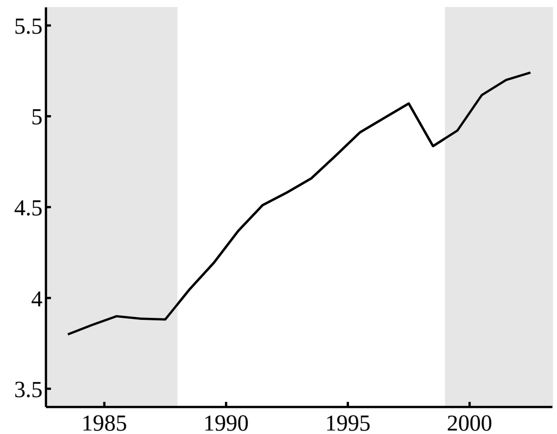
(d) Net direct and portfolio investment inflows of Indonesia (in billions of US dollars, annual moving average)



(e) Current account of Indonesia (in billions of US dollars, annual moving average)

(f) Bilateral real exchange rate between Indonesia and the United States (CPI-based, in logarithms)

Figure 17: **Case study: Indonesia - 1997.**



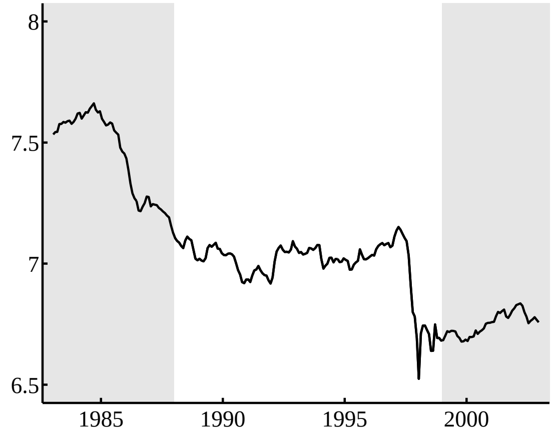
(a) Private consumption volume of Malaysia (in logarithms)

(b) Investment volume of Malaysia (in logarithms)

(c) GDP volume of Malaysia (GDP 2005 = 100, in logarithms)

(d) Net direct and portfolio investment inflows of Malaysia (in billions of US dollars)

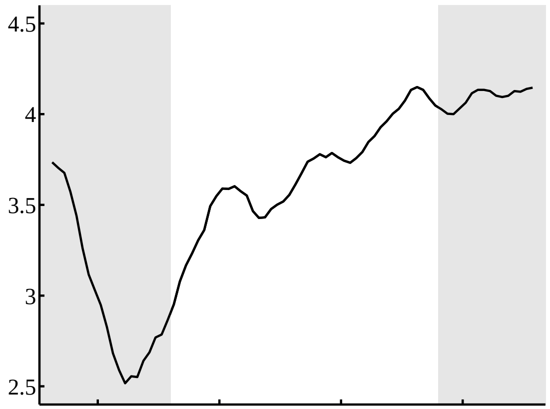
(e) Current account of Malaysia (in billions of US dollars)



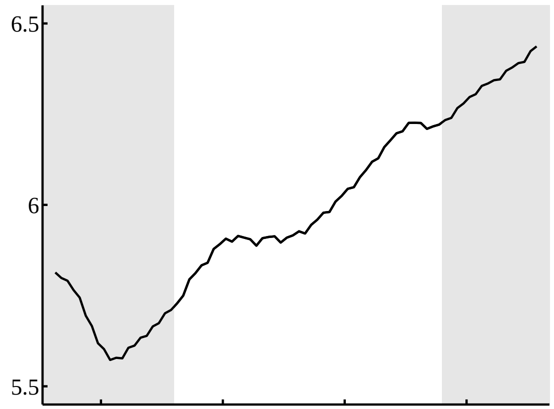
(f) Real effective exchange rate of Malaysia (CPI-based, in logarithms)

Figure 18: **Case study: Malaysia - 1997.**

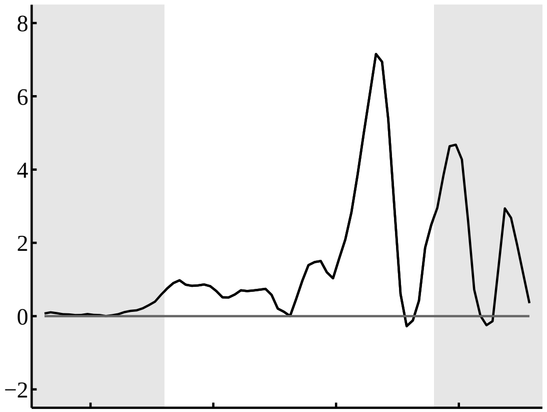
(a) Private consumption volume of Philippines (in logarithms, annual moving average)



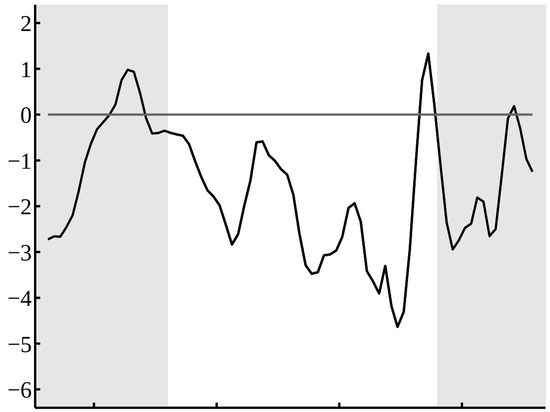
(b) Investment volume of the Philippines (in logarithms, annual moving average)



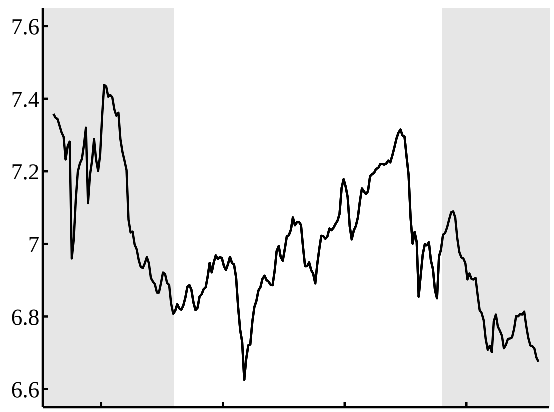
(c) GDP volume of the Philippines (GDP 2005 = 100, in logarithms, annual moving average)



(d) Net direct and portfolio investment inflows of Philippines (in billions of US dollars, annual moving average)

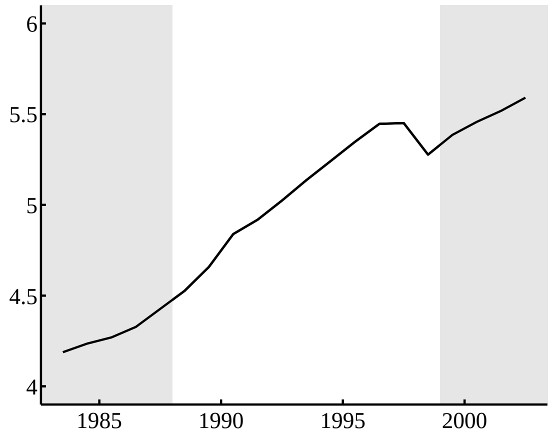


(e) Current account of Philippines (in billions of US dollars, annual moving average)

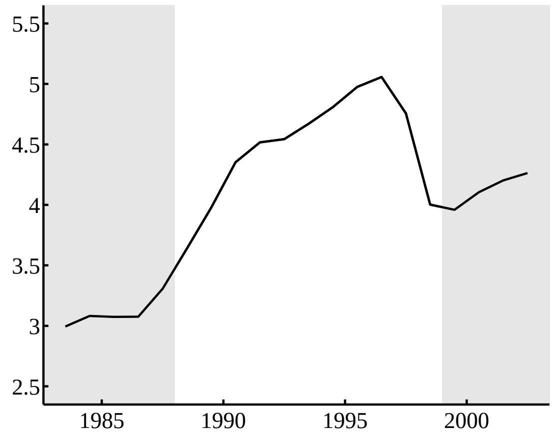


(f) Real effective exchange rate of Philippines (CPI-based, in logarithms)

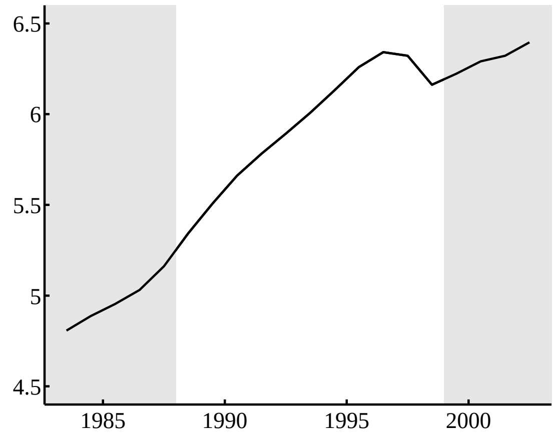
Figure 19: **Case study: Philippines - 1997.**



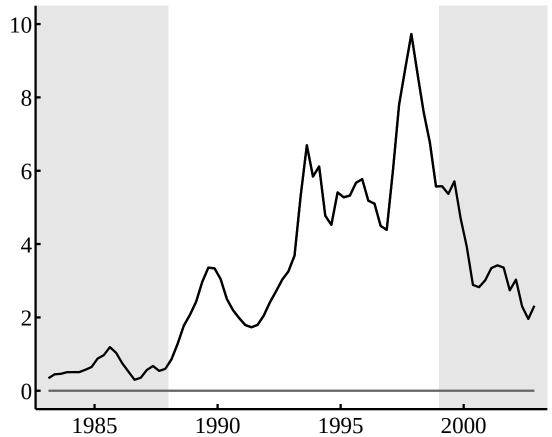
(a) Private consumption volume of Thailand (in logarithms)



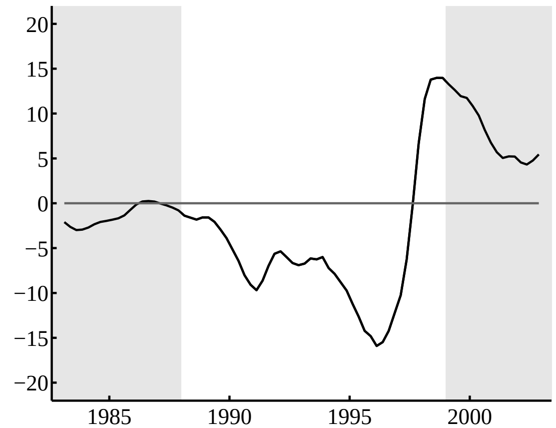
(b) Investment volume of Thailand (in logarithms)



(c) GDP volume of Thailand (GDP 2005 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of Thailand (in billions of US dollars, annual moving average)



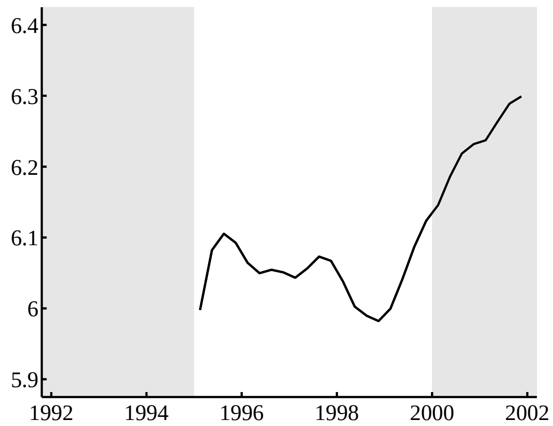
(e) Current account of Thailand (in billions of US dollars, annual moving average)

(f) Bilateral real exchange rate between Thailand and the United States (CPI-based, in logarithms)

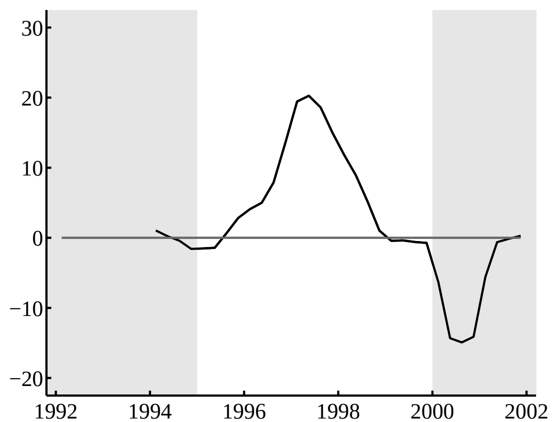
Figure 20: **Case study: Thailand - 1997.**

(a) Private consumption volume of Russia (in logarithms, annual moving average)

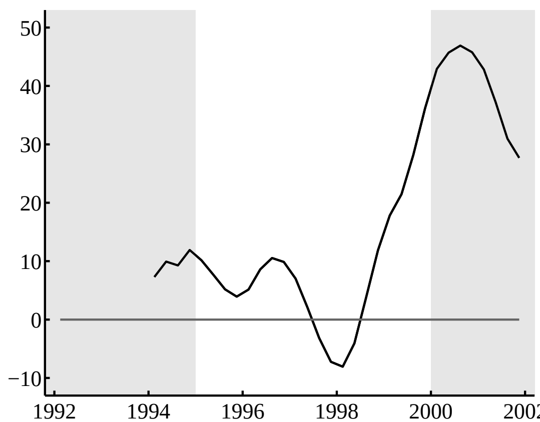
(b) Investment volume of Russia (in logarithms, annual moving average)



(c) GDP volume of Russia (GDP 2005 = 100, in logarithms, annual moving average)



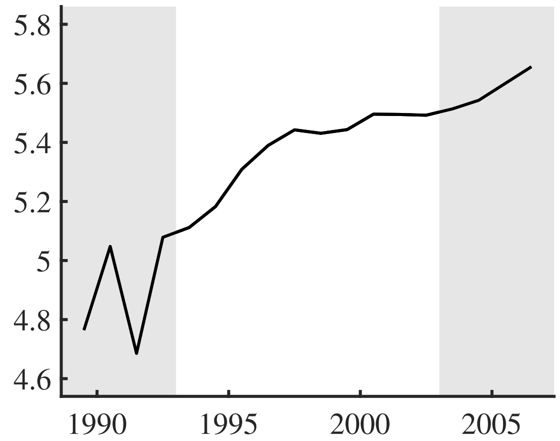
(d) Net direct and portfolio investment inflows of Russia (in billions of US dollars, annual moving average)



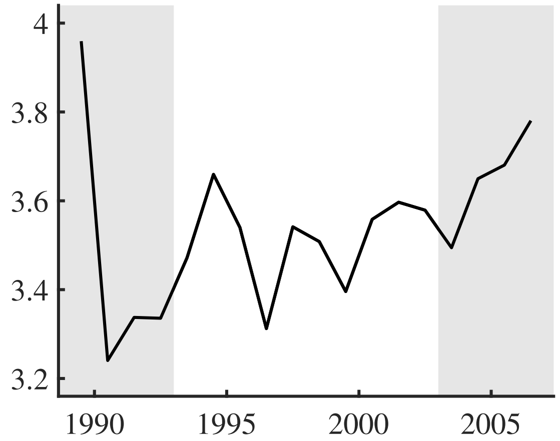
(e) Current account of Russia (in billions of US dollars, annual moving average)

(f) Real effective exchange rate of Russia (CPI-based, in logarithms)

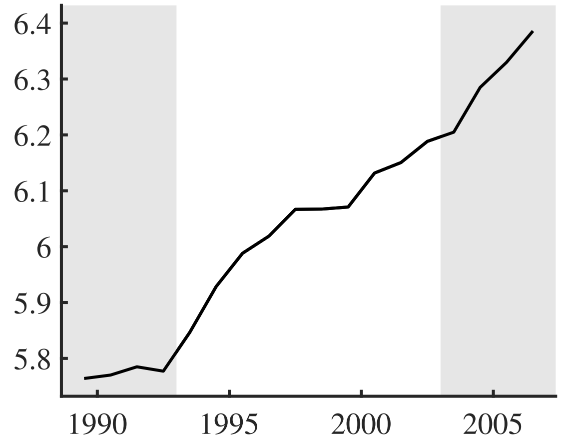
Figure 21: **Case study: Russia - 1998.**



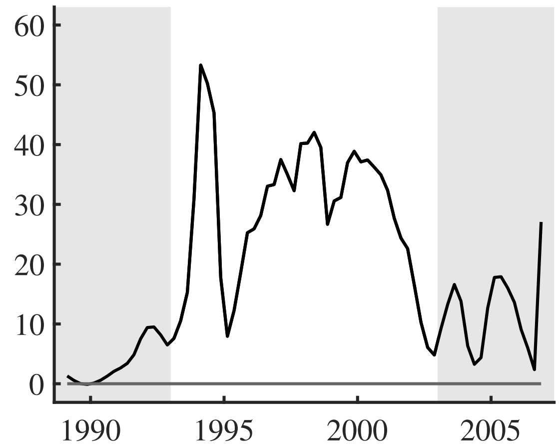
(a) Private consumption volume of Brazil (in logarithms)



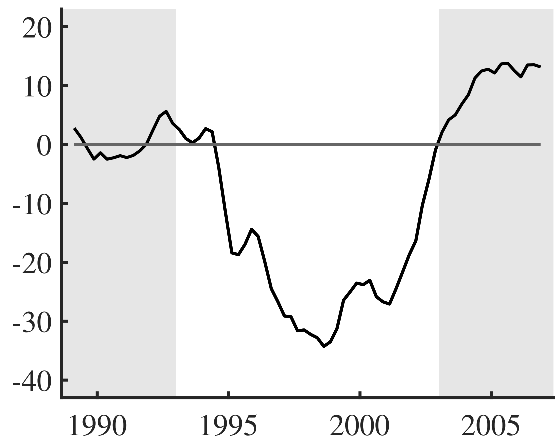
(b) Investment volume of Brazil (in logarithms)



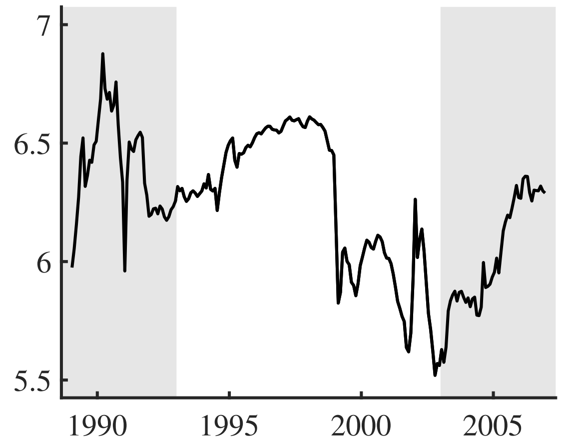
(c) GDP volume of Brazil (GDP 2010 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of Brazil (in billions of US dollars, annual moving average)



(e) Current account of Brazil (in billions of US dollars, annual moving average)

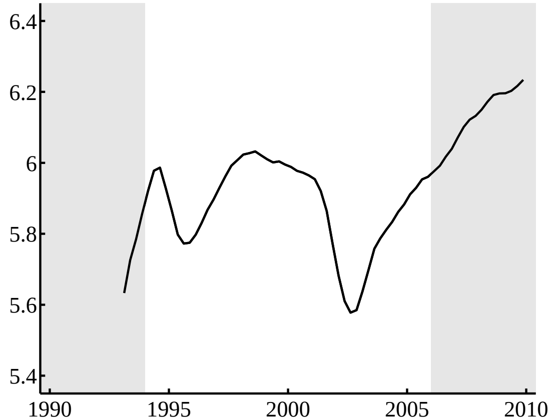


(f) Real effective exchange rate of Brazil (CPI-based, in logarithms)

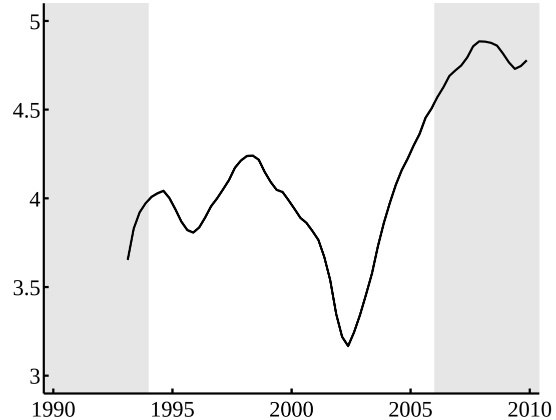
Figure 22: **Case study: Brazil - 1999.**

9.5 Argentina - 1999–2002

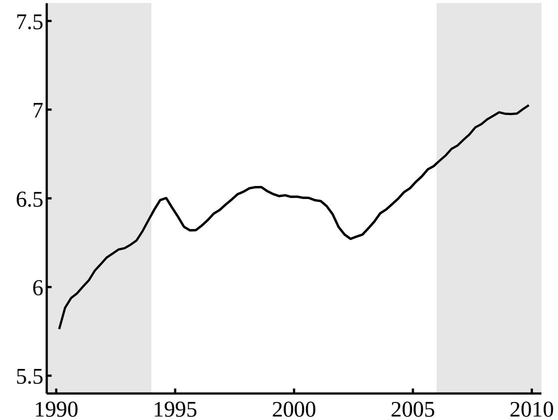
- **Boom from 1990 to 1998:** real GDP of 5.7% per year, real consumption growth of 3.8% per year, real investment growth of 10.5% per year
- **Recession from 1998 to 2002:** real GDP falling by 4.9% per year, real consumption by 7.5% per year and real investment by 16.3% per year
- **Third-highest current account deficit** in the world (of 162 countries, after the United States and Brazil)
- **Currency board from 1991 to 2002:** end of a long period of high and very high inflation in Argentina
- Increasing **economic and political cost of the exchange-rate arrangement:**
  - **fall in output** from 1999
  - **currency devaluation in Brazil,** Argentina's main trading partner, in 1999
  - **reversal of capital flows** in the early 2000s
  - **high government indebtedness**
  - emergence of **complementary currencies**



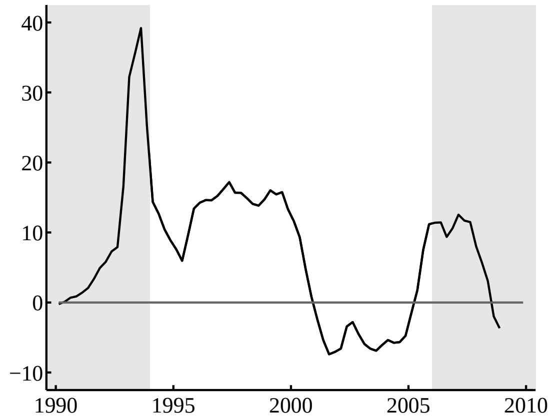
(a) Private consumption volume of Argentina (in logarithms)



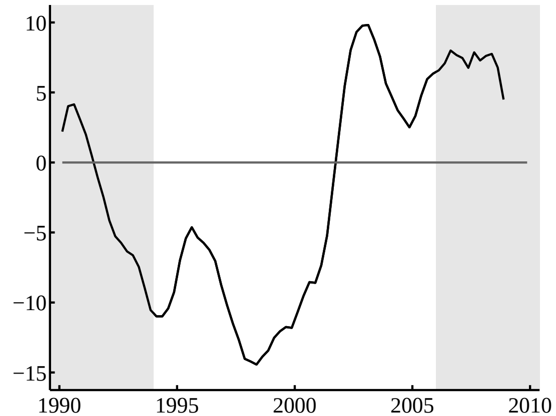
(b) Investment volume of Argentina (in logarithms)



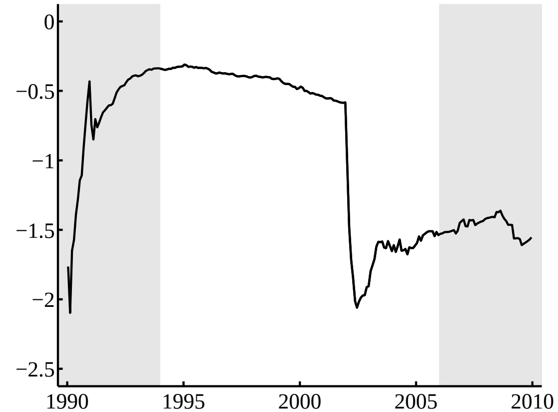
(c) GDP volume of Argentina (GDP 2005 = 100, in logarithms)



(d) Net direct and portfolio investment inflows of Argentina (in billions of US dollars, annual moving average)



(e) Current account of Argentina (in billions of US dollars)



(f) Log of bilateral real exchange rate between Argentina and the USA (in logarithms)

Figure 23: **Case study: Argentina - 1999–2002.**

10 The open economy

10.1 Review of the IS-LM model (closed economy)

10.1.1 The income-expenditure model

In the income-expenditure model, it is assumed that in equilibrium the national income of an economy equals its national spending (= aggregate demand):

$$Y^I = Y^S = Y. \tag{257}$$

The model assumes that national spending, or national expenditure, is given by consumption, which in turn is defined as:

$$Y^S = C = C^A + cY^I, \tag{258}$$

where C^A is autonomous consumption, c is the marginal propensity to consume and $0 < c < 1$. Combining both equations we get:

$$\begin{aligned} Y &= C^A + cY \\ &= \frac{1}{1-c}C^A \\ &= (1 + c + c^2 + c^3 + \dots)C^A. \end{aligned} \tag{259}$$

The main conclusion of this model is that an increase in autonomous consumption, C^A, of one unit produces a rise in income of $\frac{1}{1-c}$ units, that is, a much higher increase in income than in consumption. In fact, this result is due to a chain of effects since the initial increase in autonomous consumption of one unit causes an increase in income of one unit, too, this produces an additional increase in consumption of c units, which in turn gives rise to an increase in income of c units, causing another increase in consumption of c^2 units etc.

10.1.2 The market of goods and services

$$
\begin{aligned}
Y &= C + I + G \\
&= C^A + c(1-t)Y + I^A - dr + G^A + tY \\
&= \frac{1}{(1-c)(1-t)}(C^A + I^A - dr + G^A),
\end{aligned}
\tag{260}
$$

donde $d > 0$, $c > 0$ y $t < 1$.

The equation 260 gives rise to a curve with a negative slope in a diagram with Y on the horizontal axis and r on the vertical axis:

$$
r \downarrow \quad \Rightarrow \quad I(r) \uparrow \quad \Rightarrow \quad Y \uparrow . \tag{261}
$$

The following variables cause a shift of the IS curve (in the shown direction):

Increase of variable	IS curve
C^A	$\nearrow$
c	$\nearrow$
t	$\nearrow$
I^A	$\nearrow$
G^A	$\nearrow$

The parameter d affects the slope of the IS curve. When d rises, the slope becomes less negative.

Autonomous public spending.

Note that if public spending does not depend on income (that is, if $G = G^A$), then

$$\begin{aligned} Y &= C + I + G \\ &= C^A + c(1-t)Y + I^A - dr + G^A \\ &= \frac{1}{1-c(1-t)}(C^A + I^A - dr + G^A), \end{aligned} \tag{262}$$

and an increase in the tax rate leads to a decrease (!) of income. In this case:

Increase of variable	IS Curve
t	$\swarrow$

10.1.3 The money market

$$\frac{M}{P} = L(Y_{[+]}, (r+\pi)_{[-]}). \tag{263}$$

The left-hand side of the equation represents the real money supply, the right-hand side the real money demand. In equilibrium, real money demand and real money supply are equal. The money demand depends positively on the income (transactions motive) and negatively on the nominal interest rate (speculation motive).

The equation 263 gives rise to a curve with a positive slope in a diagram with Y on the horizontal axis and r on the vertical axis:

$$Y\uparrow \quad \Rightarrow \quad r\uparrow \quad \Rightarrow \quad \frac{M}{P} = L(\cdot,\cdot) = \text{const.} \tag{264}$$

The following variables cause a shift of the LM curve (in the shown direction):

Increase of variable	LM curve
M	$\searrow$
P	$\nwarrow$
π	$\searrow$

10.1.4 The IS-LM model in a closed economy

Two equations - two endogenous variables (Y, r). The rest are exogenous variables. The equilibrium is where the IS and LM curves cross each other.

10.1.5 Variations and options of the economic policy

Economic policies:

Policy	Measure	Effect on r	Effect on Y
Fiscal	$G \uparrow$	$r \uparrow$ (crowding out)	$Y \uparrow$ (expansionary policy)
Monetaria	$M \uparrow$	$r \downarrow$ (liquidity effect)	$Y \uparrow$ (expansionary policy)

10.2 Mundell-Fleming model

The Mundell-Fleming model is a version of the IS-LM model for the open economy.

10.2.1 Equations

The Mundell-Fleming model consists of three equations (IS, LM, PKM):

$$\begin{aligned} Y &= C(r_{[-]}) + I(r_{[-]}) + G + CA(Q_{[-]}, Y_{[-]}, Y^*_{[+]}), \\ \frac{M}{P} &= L(Y_{[+]}, r + \pi_{[-]}), \\ r &= r^*, \end{aligned} \tag{265}$$

donde

$$\begin{aligned} r &= \text{ real interest rate } = R - \pi, \\ \pi &= \text{ inflation rate.} \end{aligned} \tag{266}$$

The derivatives of the functions are as follows (they are indicated in the equations of the model by plus and minus signs):

$$
\begin{array}{lllll}
C_1 < 0, & I_1 < 0, & CA_1 < 0, & CA_2 < 0, & CA_3 > 0, \\
L_1 > 0, & L_2 < 0. & & &
\end{array}
$$

Note:

- We assume that the country is small in that it does not influence the foreign interest rate (or other foreign variables).

10.2.2 Graphic representation

The model's equations give rise to three curves relating Y (on the horizontal axis) and r (on the vertical axis):

Curve	Equation	Relationship between Y and r	Slope
IS	$Y = C(r) + I(r) + G + CA(Q, Y, Y^*)$	$Y \uparrow \Rightarrow C(r), I(r) \uparrow \Rightarrow r \downarrow$	negative
LM	$\frac{M}{P} = L(Y, r + \pi)$	$Y \uparrow \Rightarrow r \uparrow$	positive
PKM	$r = r^*$	$Y \uparrow \Rightarrow r$ constant	zero

Note:

- The IS curve represents all combinations of Y and r where domestic output, or domestic income, equals domestic expenditure.
- The LM curve represents all combinations of Y and r that ensure equilibrium in the money market.
- The curve PKM represents perfect capital mobility across borders.

10.2.3 Exogenous and endogenous variables

- three equations $\Rightarrow$ three endogenous variables
- distinguish two cases:
  - flexible exchange rate: Q endogenous
  - fixed exchange rate: Q exogenous

	Endogenous variables	Exogenous variables
Flexible exchange rate	Y, r, Q	M, G, Y^*, P, π, r^*
Fixed exchange rate	Y, r, M	Q, G, Y^*, P, π, r^*

Note:

- Fixing the exchange rate implies that monetary policy is no longer independent; instead it becomes endogenous.

10.2.4 Shifts of the three curves

Changes in the exogenous variables produce the following shifts of the three curves:

Curve	Equation	Movement	Causes
IS	$Y = C(r) + I(r) + G + CA(Q, Y, Y^*)$	$\nearrow$	$G \uparrow, Q \downarrow, Y^* \uparrow$
		$\swarrow$	$G \downarrow, Q \uparrow, Y^* \downarrow$
LM	$\frac{M}{P} = L(Y, r + \pi)$	$\searrow$	$M \uparrow, P \downarrow, \pi \uparrow$
		$\nwarrow$	$M \downarrow, P \uparrow, \pi \downarrow$
PKM	$r = r^*$	$\uparrow$	$r^* \uparrow$
		$\downarrow$	$r^* \downarrow$

10.2.5 Equilibrium

In equilibrium, the three curves cross each other. If they don't cross each other, there will be an automatic adjustment:

- If the exchange rate is flexible, the variable Q will adapt itself, shifting the IS curve in the necessary direction to achieve overall equilibrium.
- If the exchange rate is fixed, the variable M will adapt itself, shifting the LM curve in the necessary direction to achieve overall equilibrium.

10.3 Fiscal and monetary policy in the Mundell-Fleming model

The following table summarizes the consequences of fiscal and monetary policy in the Mundell-Fleming model:

	IS-LM model		Mundell-Fleming model			
Exchange rate			flexible		fixed	
Policy	$G\uparrow$	$M\uparrow$	$G\uparrow$	$M\uparrow$	$G\uparrow$	$M\uparrow$
Initial effect	IS $\nearrow$	LM $\searrow$	IS $\nearrow$	LM $\searrow$	IS $\nearrow$	LM $\searrow$
Endogenous variable			Q	Q	M	M
Disequilibrium			IS	IS	LM	LM
Endogenous adjustment			$Q\uparrow$	$Q\downarrow$	$M\uparrow$	$M\downarrow$
Secondary effect			IS $\swarrow$	IS $\nearrow$	LM $\searrow$	LM $\nwarrow$
Efficiency of policy	$Y\uparrow$	$Y\uparrow$	—	$Y\uparrow(2\times)$	$Y\uparrow(2\times)$	—
Interest rate	$r\uparrow$	$r\downarrow$	$r=r^*$	$r=r^*$	$r=r^*$	$r=r^*$

Results:

- Fiscal policy is very efficient (more than in a closed economy) when the exchange rate is fixed (liquidity effect) but loses its efficiency when the exchange rate is flexible.
- Monetary policy is very efficient (more than in a closed economy) when the exchange rate is flexible (competitive devaluation) but loses its efficiency when the exchange rate is fixed.

10.4 The Mundell-Fleming model for a large country

Let us now consider the case of a large country (in economic terms). We may study the consequences of this change in a model with two countries, a home country and a foreign country. The equations of the model are as follows (IS, IS*, LM, LM*, PKM):

$$\begin{aligned} Y &= C(r) + I(r) + G + CA(Q, Y, Y^*), \\ Y^* &= C(r^*) + I(r^*) + G^* - CA(Q, Y, Y^*), \\ \frac{M}{P} &= L(Y, r + \pi), \\ \frac{M^*}{P^*} &= L(Y^*, r^* + \pi^*), \\ r &= r^*. \end{aligned} \tag{267}$$

	Endogenous variables	Exogenous variables
Flexible exchange rate	Y, Y^*, r, r^*, Q	$M, M^*, G, G^*, P, P^*, \pi, \pi^*$
Fixed exchange rate	Y, Y^*, r, r^*, M	$Q, M^*, G, G^*, P, P^*, \pi, \pi^*$

Now we have two diagrams, one for the home country and one for the foreign country, each one with three curves.

The diagram of the home country has three curves relating r (vertical axis) with Y (horizontal axis):

Curve	Equation	Relationship between Y and r	Slope
IS	$Y = C(r) + I(r) + G + CA(Q, Y, Y^*)$	$Y \uparrow \Rightarrow C(r), I(r) \uparrow \Rightarrow r \downarrow$	negative
LM	$\frac{M}{P} = L(Y, r + \pi)$	$Y \uparrow \Rightarrow r \uparrow$	positive
PKM	$r = r^*$	$Y \uparrow \Rightarrow r$ constant	zero

The diagram of the foreign country also has three curves relating Y^* (horizontal axis) with r^* (vertical axis):

Curve	Equation	Relationship between Y and r	Slope
IS*	$Y^* = C(r^*) + I(r^*) + G^* - CA(Q, Y, Y^*)$	$Y^* \uparrow \Rightarrow r^* \downarrow$	negative
LM*	$\frac{M^*}{P^*} = L(Y^*, r^* + \pi^*)$	$Y^* \uparrow \Rightarrow r^* \uparrow$	positive
PKM*	$r = r^*$	$Y^* \uparrow \Rightarrow r^*$ constant	zero

Changes in the exogenous variables produce the following shifts of the curves:

Curve	Equation	Movement	Causes
IS	$Y = C(r) + I(r) + G + CA(Q, Y, Y^*)$	$\nearrow$	$G\uparrow, Q\downarrow, Y^*\uparrow$
		$\swarrow$	$G\downarrow, Q\uparrow, Y^*\downarrow$
IS*	$Y^* = C(r^*) + I(r^*) + G^* - CA(Q, Y, Y^*)$	$\nearrow$	$G^*\uparrow, Q\uparrow, Y^*\uparrow$
		$\swarrow$	$G^*\downarrow, Q\downarrow, Y^*\downarrow$
LM	$\frac{M}{P} = L(Y, r + \pi)$	$\searrow$	$M\uparrow, P\downarrow, \pi\uparrow$
		$\nwarrow$	$M\downarrow, P\uparrow, \pi\downarrow$
LM*	$\frac{M^*}{P^*} = L(Y^*, r^* + \pi^*)$	$\searrow$	$M^*\uparrow, P^*\downarrow, \pi^*\uparrow$
		$\nwarrow$	$M^*\downarrow, P^*\uparrow, \pi^*\downarrow$
PKM/	$r = r^*$	$\uparrow$	$r\uparrow, r^*\uparrow$
PKM*		$\downarrow$	$r\downarrow, r^*\downarrow$

- Any change in Q affects both the IS and the IS* curves.
- The PKM y PKM* curves are identical in both diagrams.

Exchange rate	flexible		fixed	
Policy	$G\uparrow$	$M\uparrow$	$G\uparrow$	$M\uparrow$
Initial effect	IS $\nearrow$	LM $\searrow$	IS $\nearrow$	LM $\searrow$
Endogenous variable	Q	Q	M	M
Disequilibrium	IS	IS	LM	LM
Endogenous adjustment	$Q\uparrow$	$Q\downarrow$	$M\uparrow$	$M\downarrow$
Secondary effects	IS $\swarrow$, IS* $\nearrow$	IS $\nearrow$, IS* $\swarrow$	LM $\searrow$	LM $\nwarrow$
Policy efficiency	$Y\uparrow(0.5\times)$	$Y\uparrow(1.5\times)$	$Y\uparrow(2.0\times)$	—
Interest rate	$r, r^*\uparrow$	$r, r^*\downarrow$	—	—

10.5 Application: The appreciation of the US dollar in the 1980s

10.6 Analytical solution of the Mundell-Fleming model

Let us consider a simplified version of the Mundell-Fleming model:

$$Y = -\alpha r + G - \beta q, \tag{268}$$
$$M = \gamma Y - \delta r, \tag{269}$$
$$r = r^*. \tag{270}$$

By substituting the PKM equation into the IS and LM equations, we can simplify the model even more:

$$Y = -\alpha r^* + G - \beta q, \tag{271}$$
$$M = \gamma Y - \delta r^*. \tag{272}$$

Our objective is to obtain a solution of a model like this that allows us to evaluate the effects of changes in the exogenous variables on the endogenous variables in equilibrium.

10.6.1 General method of solving a linear macroeconomic model

A linear macroeconomic model like the Mundell-Fleming model can be written in the following way:

$$ay_1 + by_2 = ex_1 + fx_2 + gx_3, \tag{273}$$

$$cy_1 + dy_2 = hx_1 + kx_2 + lx_3, \tag{274}$$

where y_1 and y_2 are endogenous variables and x_1, x_2 and x_3 are exogenous variables.

Now writing the model in matrices:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} e & f & g \\ h & k & l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{275}$$

The inverse of the matrix of the endogenous variables is:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \Rightarrow \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

If we premultiply the model with this inverse, we obtain:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} e & f & g \\ h & k & l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{276}$$

The solution is then:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} e & f & g \\ h & k & l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{277}$$

10.6.2 Application to the Mundell-Fleming model

Suppose the exchange rate, Q, is flexible. Let us first write the Mundell-Fleming model in terms of matrices:

$$\begin{bmatrix} 1 & \beta \\ -\gamma & 0 \end{bmatrix} \begin{bmatrix} Y \\ q \end{bmatrix} = \begin{bmatrix} -\alpha & 1 & 0 \\ -\delta & 0 & -1 \end{bmatrix} \begin{bmatrix} r^* \\ G \\ M \end{bmatrix}. \tag{278}$$

We now premultiply the model with the inverse of the matrix of the endogenous variables:

$$\begin{bmatrix} 1 & \beta \\ -\gamma & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \beta \\ -\gamma & 0 \end{bmatrix} \begin{bmatrix} Y \\ q \end{bmatrix} = \begin{bmatrix} 1 & \beta \\ -\gamma & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\alpha & 1 & 0 \\ -\delta & 0 & -1 \end{bmatrix} \begin{bmatrix} r^* \\ G \\ M \end{bmatrix}. \tag{279}$$

By applying the formula of the inverse of a 2×2 matrix, we obtain the solution:

$$\begin{aligned}
\begin{bmatrix} Y \\ q \end{bmatrix} &= \frac{1}{\beta\gamma} \begin{bmatrix} 0 & -\beta \\ \gamma & 1 \end{bmatrix} \begin{bmatrix} -\alpha & 1 & 0 \\ -\delta & 0 & -1 \end{bmatrix} \begin{bmatrix} r^* \\ G \\ M \end{bmatrix} \\
&= \frac{1}{\beta\gamma} \begin{bmatrix} \beta\delta & 0 & \beta \\ -\alpha\gamma - \delta & \gamma & -1 \end{bmatrix} \begin{bmatrix} r^* \\ G \\ M \end{bmatrix}.
\end{aligned} \tag{280}$$

The final solution is:

$$Y = \frac{\delta}{\gamma} r^* + \frac{1}{\gamma} M, \tag{281}$$

$$q = -\frac{\alpha\gamma + \delta}{\beta\gamma} r^* + \frac{1}{\beta} G - \frac{1}{\beta\gamma} M. \tag{282}$$

Note that fiscal policy has no effect on output while monetary policy does.

Contents

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